Renormalization Proof for Spontaneously broken Yang-Mills Theory with Flow Equations

Christoph Kopper
Centre de Physique Théorique de l'Ecole Polytechnique
F-91128 Palaiseau, France

Volkhard F. Müller
Fachbereich Physik, Universität Kaiserslautern
D-67653 Kaiserslautern, Germany

Abstract

Abstract: In this paper we present a renormalizability proof for spontaneously broken $SU(2)$ gauge theory. It is based on Flow Equations, i.e. on the Wilson renormalization group adapted to perturbation theory. The power counting part of the proof, which is conceptually and technically simple, follows the same lines as that for any other renormalizable theory. The main difficulty stems from the fact that the regularization violates gauge invariance. We prove that there exists a class of renormalization conditions such that the renormalized Green functions satisfy the Slavnov-Taylor identities of $SU(2)$ Yang-Mills theory on which the gauge invariance of the renormalized theory is based.

1 Introduction

In the early seventies Wilson and his collaborators published their ideas on the renormalization group and effective Lagrangians [WiKo], which have stimulated the progress of quantum field theory and statistical mechanics ever since. In 1984 Polchinski [Pol] showed that these ideas are suited as a basis for perturbative renormalization theory.[1] He proved Euclidean massive Φ_4^4 to be renormalizable without introducing Feynman diagrams, thus sidestepping the associated complicated analysis of their divergence/convergence properties. Instead, the problem is solved by bounding inductively the solutions of a system of first order differential equations, the Flow Equations (FE), which are a reduction of the Wilson FE to their perturbative content.
Over the past decade Polchinski's argument has been considerably simplified technically, extended to physical renormalization conditions and has been rendered rigorous [KKSa]. Beyond it has been applied, again in mathematical rigour, to nearly all situations of physical interest: The Φ_4^4 proof itself already also holds for any other massive theory with global symmetries only and renormalizable power counting, like e.g. the Yukawa-models, O(N)-models etc. It could then be extended to Euclidean massless Φ_4^4 [KK1] and QED_4 [KK2] and also to theories in Minkowski-space [KKSc]. The FE method also served to extract properties of, or bounds on Green functions which were harder - if at all - to get by other methods. We mention composite operator renormalization together with (generalized) Zimmermann identities [KK3], Wilson's operator product expansion [KK4], Symanzik improvement in the convergence of the regularized theory [Ke1, Wie], de Calan-Rivasseau large order bounds on perturbation theory [Ke2], bounds on the singularities of Green functions at exceptional momenta [KK1], analyticity properties of Green functions in Minkowski space [KKSc] and decoupling theorems [Kim]. A recent review (in German) on previous work on FEs can be found in [Kop]. We should also mention that the interest in FEs over the last decade goes far beyond mathematical physics and has led to many interesting results, ideas and calculations in theoretical physics. To give few examples we mention that critical exponents for Φ_4^4-type theories have been calculated in [TeWe]. Truncated FE have also been applied to the bound state problem in [Ell], to Yang-Mills theory in [EHW], in particular to the study of vacuum condensates in [ReWe].

Among the entries in our list on solved renormalization problems there is still one missing, which is of fundamental importance in physics, namely nonabelian gauge theory. The present paper is intended to close this gap by treating spontaneously broken SU(2)-Yang-Mills theory,

[1] Wilson himself remarked already in the late sixties that this should be possible, as we learned from E. Brézin.

which corresponds to the weak sector of the standard model.[2] Another interesting problem, which should be studied, is QCD where the problem of gauge invariance is intertwined with the infrared problem. Since the latter has already been extensively studied we chose the spontaneously broken theory which is infrared finite and thus simpler. On the other hand the Slavnov-Taylor identities (STI) or Ward identities of the spontaneously broken symmetry are more complicated to analyse.[3]

The (ultraviolet) power counting part of the FE renormalization proof is (up to notational and other minor changes) the same and simple for all the above mentioned theories, which renders the method attractive. Gauge theories, however, present a difficulty coming from the wellknown fact that gauge symmetry is broken by cutoffs in momentum space, and it is just the flow of such a cutoff which produces the FE. What we have to show is that gauge invariance is restored when the cutoffs are taken away. On the level of the Green functions (which are not gauge invariant) this means that we have to verify the STI of the theory. They then allow to argue that physical quantities such as the S-matrix are gauge-invariant[ZiJ]. On analysing the FE for a gauge theory one realizes that the restoration of the STI depends on the choice of the renormalization conditions chosen and cannot be true in general. More precisely, since gauge invariance is violated in the regularized theory, the renormalization group flow will generally produce nonvanishing contributions to all those relevant parameters of the theory, which are forbidden by gauge invariance, e.g. a noninvariant gauge field selfcoupling of the form $(\vec{A}^2)^2$. The question is then: Can we use the freedom in adjusting the renormalization conditions such that the STI are nevertheless restored in the end? To answer this question a first observation, already encountered when treating QED, is crucial: The violation of the STI in the regularized theory can be expressed through Green functions carrying an operator insertion, which depends on the regulators. FE theory for such insertions tells us that these Green functions will vanish once the cutoffs are removed, if we achieve renormalization conditions on the theory such that the inserted Green functions (uniquely calculated from those) have vanishing renormalization conditions for all relevant terms, i.e. up to the dimension of the insertion (which is 5 in our case). Comparing the number of relevant terms for the SU(2) theory - 37 (see App.A)- and for the insertion - 53 (see App.C)-, we realize that it is not possible to make vanish 53 terms on adjusting 37 free parameters, unless there are linear interdependences. It is again the FE

[2] for vanishing Weinberg angle. This is however not of decisive importance for the line of the argument. It matters insofar as the explicit description and treatment of the whole SU(2)×U(1)-theory would require much more space.

[3] We mention also that FE and STI for pure Yang-Mills theory in the limit case without UV cutoff have been considered in [BAM2].

(in its global integrated form) which helps us to make transparent these interdependences. The problem of how to find one's way through the STI and adjusting the renormalization conditions appropriately is somewhat complicated through spontaneous symmetry breaking, since the latter mixes Green functions of different dimension.

One may of course ask the question whether such a proof of the renormalizability of Yang-Mills theory is still necessary in view of the fact that the problem has been settled in the seventies by the pioneering work of 't Hooft and Veltman and successors. Without going into details or giving references on work which has made entrance into nearly all textbooks on quantum field theory or particle physics we would still like to mention that there rests a bit of uneasiness on the mathematical physicists' side on the form in which the subject has settled in the course of time. This is because the standard way in which the argument is presented nowadays is based on two main ingredients: the existence of an invariant regularization scheme, i.e. dimensional regularization, and algebraic manipulations on generating functionals, which can be given rigorous meaning for regularized path integral formulations. To date nobody has achieved a (rigorous) definition of dimensionally regularized path integrals so that there remains a gap in the reasoning which could only be closed if the analysis of the STI were directly performed on individual Feynman graphs, a presumably awkward procedure. These arguments do not apply to the lattice regularization [4], which allows for a (particularly transparent) path integral formulation while respecting gauge invariance. It violates Euclidean or Lorentz symmetry however. We emphasize the work of Reiß as a largely coherent and rigorous analysis of the perturbative renormalization problem of (QCD type) gauge theories on the lattice [Rei]. His work is based on an adaptation of BPHZ renormalization to the lattice, where quite a number of new problems appear.

As a guide to the logical structure of the paper we now expose the main line of arguments. Our starting point is a massive UV regularized theory. The generating functional L^{Λ,Λ_0} of the connected amputated Green functions (CAG) with momenta in the interval $[\Lambda,\Lambda_0]$ satisfies a flow equation (35) with respect to Λ, which when reduced to its perturbative content (37) permits to bound inductively the l-loop n-point functions $\mathcal{L}_{l,n}^{\Lambda,\Lambda_0}$ in such a way (39, 43) that their existence for $\Lambda_0 \to \infty$ becomes obvious. This is true for all theories renormalizable by power counting under the condition that all relevant terms, i.e. local terms of mass dimension ≤ 4 are fixed by (Λ_0-independent) renormalization conditions (r.c.). In gauge theories the number of such terms is generally much bigger than the number of free parameters of the theory. For our model the respective numbers are 37 (listed in App.A) and 8 (cf. (121)).

[4] the above mentioned algebraic analysis is however based on the continuum formulation.

So most of the r.c. cannot be freely chosen for a gauge theory. A priori it does not seem possible to guess which r.c. are the right ones.

Thus we analyse the action L^{0,Λ_0} for general r.c. and expose the violation of the STI as a functional associated with an operator insertion, which turns out to be of dimension 5. We denote it as $L_1 = L_1^{0,\Lambda_0}$ (75). This is achieved on using an UV regularized version (62, 66) of the BRS transformation (13, 14, 18). General results from FE theory tell us that L_1^{0,Λ_0} will vanish for $\Lambda_0 \to \infty$ if all its relevant terms, i.e. the local parts of dimension ≤ 5, are fixed to be 0 by the r.c. and if the irrelevant terms in $L_1^{\Lambda_0,\Lambda_0}$ vanish sufficiently rapidly for $\Lambda_0 \to \infty$ (110). The 53 renormalization parts for L_1^{0,Λ_0} (see App.C) are functions of the 37 r.c. for L^{0,Λ_0} and 7 free parameters in the BRS transformation (see App.B). Thus if the model can be renormalized respecting the STI there must be linear interdependences among the 53 relations. These are not explicit in the theory L^{0,Λ_0}, since L^{0,Λ_0} contains irrelevant terms of arbitrary dimension which are not known explicitly. We therefore derive the violated Slavnov-Taylor identities (VSTI) also in terms of the bare functionals L^{Λ_0,Λ_0} and $L_1^{\Lambda_0,\Lambda_0}$ (98, 99), using again the FE for that purpose. The FE may also be used (104, 113-120) to relate L_1^{0,Λ_0} and $L_1^{\Lambda_0,\Lambda_0}$ with each other (111, 112) so that - respecting the inductive procedure, i.e. climbing up in the loop order l, and for given l in the number of external legs n - we may hope to satisfy the STI (for $\Lambda_0 \to \infty$) as well by imposing the relevant terms in $L_1^{\Lambda_0,\Lambda_0}$ to vanish (instead of those in L_1^{0,Λ_0}). Since L^{Λ_0,Λ_0} does not contain unknown[5] irrelevant terms an explicit analysis of the bare STI is possible, and we can make vanish 53 terms order by order in l on appropriately fixing L^{Λ_0,Λ_0} and the free BRS constants. However starting at the wrong end - i.e. fixing counter terms instead of r.c. - we cannot prove renormalizability. Thus the task is threefold :

i) Reveal a number of free renormalization constants corresponding to the free parameters of the theory (121).

ii) Satisfy a subset of the STI for the relevant parts by choosing appropriate r.c. for L^{0,Λ_0} (125, 127). This subset has to be chosen sufficiently large to get hold on the finiteness problem, with the help of the FE and afterwards also of the STI themselves.

iii) Satisfy the remaining STI for the relevant parts by choosing the appropriate l-loop terms in L^{Λ_0,Λ_0} (122, 123, 124). It is possible indeed to show that all remaining STI ((128, 129, 130) and those mentioned after (131)) can be satisfied. These are far more than the constants fixed in iii). All this has to be done respecting the order of the inductive procedure.

If it were not possible to make ends meet (i.e. if either the subset in ii) is too small to prove finiteness, or the one in iii) is too small in order to satisfy all STI) we would face what is

[5] L^{Λ_0,Λ_0} will include some well-behaved irrelevant terms (107, 108) linked to the particular nature of the cutoff (30) chosen.

called an anomaly.

Our procedure is complicated by a technical point. The analysis of the relevant part of the STI at $\Lambda = 0$ is much more complicated for L^{0,Λ_0} than for Γ^{0,Λ_0}, the generating functional of the one-particle irreducible functions. For L^{0,Λ_0} many more terms of the same loop order may appear in a single STI. Passing to one-particle irreducible objects achieves to a considerable degree the disentangling of the l-loop renormalization parts in the inhomogeneous linear equations of App.C. So App.C has indeed been written for the Γ^{0,Λ_0}- and not for the L^{0,Λ_0}-functional. The price to pay is that we have to provide for the necessary machinery for the Γ-functional (flow equations (87), STI (82)) too, using the Legendre transform (78, 79). This should not obscure the fact that all results of this paper are to be obtained from L^{0,Λ_0}.

This paper is organized as follows. In chapter 2 we introduce the classical action of the model and fix notations. In chapter 3 we introduce the concepts from FE theory and recall the statements on renormalizability we need. As regards the general aspects on bounding inductively solutions of the FE we tend to be short as long as the reasoning follows the lines of previous papers. In chapter 4 we derive the VSTI for the regularized theory in various forms, comment on the adaptation of the renormalization results to the vertex functions, analyse the above mentioned operator insertion and show how to make vanish its relevant parts step by step on disposing of the freedom in choosing the renormalization conditions. This is the key part of the paper. With the aid of the results from chapter 3 it permits to prove that the STI are restored and thus solves the renormalization problem for spontaneously broken SU(2) Yang-Mills theory.

2 Classical theory and Tree approximation

We collect some basic properties of the classical Euclidean SU(2)- Yang-Mills-Higgs model in four dimensional Euclidean spacetime, mainly to introduce the notation and the conventions. We largely follow the textbook of Faddeev and Slavnov [FaSl].

The action considered involves the real Yang-Mills field $\{A^a_\mu\}_{a=1,2,3}$ and the complex scalar doublet $\{\phi_\alpha\}_{\alpha=1,2}$. All bosonic fields appearing in this paper may be viewed as smooth functions of (sufficiently) rapid fall-off. Details do not matter in view of the fact that we do not perform any nonperturbative analysis of path integrals. The action has the form

$$S_{\rm inv} = \int dx \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} (\nabla_\mu \phi)^* \nabla_\mu \phi + \lambda (\phi^* \phi - \rho^2)^2 \right\}, \tag{1}$$

with the curvature tensor

$$F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g\epsilon^{abc} A^b_\mu(x) A^c_\nu(x) \tag{2}$$

and the covariant derivative

$$\nabla_\mu = \partial_\mu + g\frac{1}{2i}\sigma^a A^a_\mu(x) \tag{3}$$

acting on the SU(2)-spinor ϕ. The parameters g, λ, ρ are real positive, ϵ^{abc} is totally skew symmetric, $\epsilon^{123} = +1$, and $\{\sigma^a\}_{a=1,2,3}$ are the standard Pauli matrices. For simplicity the wave function normalizations of the fields are chosen equal to one. The action (1) is invariant under local gauge transformations of the fields

$$\frac{1}{2i}\sigma^a A^a_\mu(x) \longrightarrow u(x)\frac{1}{2i}\sigma^a A^a_\mu(x) u^*(x) + g^{-1} u(x)\partial_\mu u^*(x)\,, \quad \phi(x) \longrightarrow u(x)\phi(x) \tag{4}$$

with $u : \mathrm{I\!R}^4 \to \mathrm{SU}(2)$ smooth. A stable ground state of the action (1) implies spontaneous symmetry breaking, taken into account by reparametrizing the complex scalar doublet as

$$\phi(x) = \begin{pmatrix} B^2(x) + iB^1(x) \\ \rho + h(x) - iB^3(x) \end{pmatrix}\,, \tag{5}$$

where $\{B^a(x)\}_{a=1,2,3}$ is a real triplet and $h(x)$ the real Higgs field. Moreover, in place of the parameters ρ, λ we introduce the masses

$$m = \frac{1}{2}g\rho, \quad M = (8\lambda\rho^2)^{\frac{1}{2}}. \tag{6}$$

Aiming at a quantized theory we choose the 't Hooft gauge fixing

$$S_{\mathrm{g.f.}} = \int dx\, \frac{1}{2\alpha}(\partial_\mu A^a_\mu - \alpha m B^a)^2, \tag{7}$$

with $\alpha \in \mathrm{I\!R}_+$, implemented by anticommuting Faddeev-Popov ghost and antighost fields $\{c^a\}_{a=1,2,3}$ and $\{\bar{c}^a\}_{a=1,2,3}$, respectively, via

$$S_{\mathrm{gh}} = -\int dx \bar{c}^a \{(-\partial_\mu\partial_\mu + \alpha m^2)\delta^{ab} + \frac{1}{2}\alpha g m\, h\delta^{ab} + \frac{1}{2}\alpha g m \epsilon^{acb} B^c - g\partial_\mu \epsilon^{acb} A^c_\mu\} c^b. \tag{8}$$

Hence, the total "classical action" is

$$S_{\mathrm{BRS}} = S_{\mathrm{inv}} + S_{\mathrm{g.f.}} + S_{\mathrm{gh}}, \tag{9a}$$

which we decompose as

$$S_{\mathrm{BRS}} = \int dx\, \{\mathcal{L}_{\mathrm{quad}}(x) + \mathcal{L}_{\mathrm{int}}(x)\} \tag{9b}$$

into its quadratic part, with $\Delta \equiv \partial_\mu \partial_\mu$,

$$\begin{aligned}\mathcal{L}_{\text{quad}} &= \frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + \frac{1}{2\alpha}(\partial_\mu A^a_\mu)^2 + \frac{1}{2}m^2 A^a_\mu A^a_\mu \\ &+\frac{1}{2}h(-\Delta + M^2)h + \frac{1}{2}B^a(-\Delta + \alpha m^2)B^a \\ &-\bar{c}^a(-\Delta + \alpha m^2)c^a\,, \end{aligned} \tag{10}$$

and into its interaction part

$$\begin{aligned}\mathcal{L}_{\text{int}} &= g\epsilon^{abc}(\partial_\mu A^a_\nu)A^b_\mu A^c_\nu + \frac{1}{4}g^2(\epsilon^{abc}A^b_\mu A^c_\nu)^2 \\ &+\frac{1}{2}g\left\{(\partial_\mu h)A^a_\mu B^a - hA^a_\mu\partial_\mu B^a - \epsilon^{abc}A^a_\mu(\partial_\mu B^b)B^c\right\} \\ &+\frac{1}{8}gA^a_\mu A^a_\mu\left\{4mh + g(h^2 + B^a B^a)\right\} \\ &+\frac{1}{4}g\frac{M^2}{m}\,h(h^2 + B^a B^a) + \frac{1}{32}g^2\left(\frac{M}{m}\right)^2(h^2 + B^a B^a)^2 \\ &-\frac{1}{2}\alpha g m\bar{c}^a\left\{h\delta^{ab} + \epsilon^{acb}B^c\right\}c^b \\ &-g\epsilon^{acb}(\partial_\mu\bar{c}^a)A^c_\mu c^b. \end{aligned} \tag{11}$$

In (10) we recognize that all fields are massive and that no coupling term $A^a_\mu\partial_\mu B^a$ appears. The propagators of the Yang-Mills field A^a_μ, of the Higgs field h, and of the ghost field c^a and the Goldstone field B^a, are thus (respectively)

$$C^{ab}_{\mu\nu}(p) = \frac{\delta^{ab}}{p^2+m^2}\{\delta_{\mu\nu} - (1-\alpha)\frac{p_\mu p_\nu}{p^2+\alpha m^2}\}\,,\; C(p) = \frac{1}{p^2+M^2}\,,\; S^{ab}(p) = \frac{\delta^{ab}}{p^2+\alpha m^2}\,. \tag{12}$$

The classical action S_{BRS} in (9b) has the following properties

i) Euclidean invariance: S_{BRS} is an O(4)-scalar.

ii) Rigid SO(3)-isosymmetry: The fields $\{A^a_\mu\}, \{B^a\}, \{c^a\}, \{\bar{c}^a\}$ are isovectors and h is an isoscalar; S_{BRS} is invariant under global SO(3)-transformations.

iii) BRS-invariance: Introducing the classical composite fields

$$\psi^a_\mu(x) = \left\{\partial_\mu\delta^{ab} + g\epsilon^{arb}A^r_\mu(x)\right\}c^b(x), \tag{13a}$$

$$\psi(x) = -\frac{1}{2}gB^a(x)c^a(x), \tag{13b}$$

$$\psi^a(x) = \left\{(m + \frac{1}{2}g\,h(x))\delta^{ab} + \frac{1}{2}g\epsilon^{arb}B^r(x)\right\}c^b(x), \tag{13c}$$

$$\Omega^a(x) = \frac{1}{2}g\epsilon^{apq}c^p(x)c^q(x)\,, \tag{13d}$$

the BRS-transformations of the fields are defined as

$$A^a_\mu(x) \longrightarrow A^a_\mu(x) - \psi^a_\mu(x)\epsilon, \tag{14a}$$

$$h(x) \longrightarrow h(x) - \psi(x)\epsilon, \tag{14b}$$

$$B^a(x) \longrightarrow B^a(x) - \psi^a(x)\epsilon, \tag{14c}$$

$$c^a(x) \longrightarrow c^a(x) - \Omega^a(x)\epsilon, \tag{14d}$$

$$\bar{c}^a(x) \longrightarrow \bar{c}^a(x) - \frac{1}{\alpha}(\partial_\nu A^a_\nu(x) - \alpha m B^a(x))\epsilon\,. \tag{14e}$$

In these transformations ϵ is a spacetime independent Grassmann element that commutes with the fields $\{A^a_\mu, h, B^a\}$ but anticommutes with the (anti-)ghosts $\{c^a, \bar{c}^a\}$. To show the BRS-invariance of the total classical action (9) one first observes that the composite classical fields (13) are themselves invariant under the BRS-transformations (14). Moreover, we can write (8) in the form

$$S_{\mathrm{gh}} = -\int dx \bar{c}^a\{-\partial_\mu\psi^a_\mu + \alpha m\psi^a\}. \tag{15}$$

Using these properties the BRS-invariance of (9) is straightforward (if somewhat tedious) to verify.

It is convenient to add to the classical action (9) source terms both for the fields and the composite fields (13), defining

$$S_c = S_{\mathrm{BRS}} + \int dx\{\gamma^a_\mu\psi^a_\mu + \gamma\psi + \gamma^a\psi^a + \omega^a\Omega^a\} - \int dx\{j^a_\mu A^a_\mu + sh + b^a B^a + \bar{\eta}^a c^a + \bar{c}^a\eta^a\}. \tag{16}$$

The sources $\gamma^a_\mu, \gamma, \gamma^a$ have dimension 2, ghost number -1 and are Grassmann elements, whereas ω^a has dimension 2 and ghost number -2; the sources η^a and $\bar{\eta}^a$ have ghost number +1 and -1, respectively, and are Grassmann elements. The BRS-transformation (14) of S_c can be written as

$$S_c \longrightarrow S_c + \mathcal{D}S_c\epsilon \tag{17}$$

employing the BRS-operator $\mathcal{D}$, defined by

$$\mathcal{D} = \int dx \left\{ j^a_\mu \frac{\delta}{\delta\gamma^a_\mu} + s\frac{\delta}{\delta\gamma} + b^a\frac{\delta}{\delta\gamma^a} + \bar{\eta}^a\frac{\delta}{\delta\omega^a} + \eta^a\left(\frac{1}{\alpha}\partial_\nu\frac{\delta}{\delta j^a_\nu} - m\frac{\delta}{\delta b^a}\right)\right\}\,. \tag{18}$$

(Observe that ϵ anticommutes with $\eta, \bar{\eta}$, too.)

For some purposes it will turn out convenient to regard the fields and functionals thereof in momentum space. Our conventions are

$$\phi(x) = \int_p e^{ipx}\,\hat{\phi}(p)\,, \quad \int_p = \int \frac{d^4p}{(2\pi)^4}\,, \tag{19}$$

where mostly we will omit the hat on $\phi(p)$. From (19) we obtain

$$\frac{\delta}{\delta\phi(x)} = \int d^4p \; e^{-ipx} \frac{\delta}{\delta\hat{\phi}(p)} = (2\pi)^4 \int_p e^{-ipx} \frac{\delta}{\delta\hat{\phi}(p)} \; .$$

For functionals with operator insertions like e.g.

$$S_{\gamma(x)} \; := \frac{\delta S_c}{\delta\gamma(x)} \quad \text{we define } S_{\gamma(p)} \; := \int d^4x \; e^{ipx} \, S_{\gamma(x)} \tag{20}$$

(again in abusively shortened notation). For later use it will be convenient to introduce a shortened collective notation for the fields, sources and propagators. As for the latter, we will sometimes denote all propagators (12) collectively by C. Furthermore we write

$$\text{for the bosonic fields } \varphi_\tau = (A^a_\mu, \; h, \; B^a) \; \text{ with corresponding sources } \; J_\tau = (j^a_\mu, \; s, \; b^a\,) \, , \tag{21}$$

$$\text{for all fields } \; \Phi = (\varphi_\tau, \; c^a, \; \bar{c}^a) \; \text{ and for their sources } K = (J_\tau, \; \overline{\eta}^a, \; \eta^a) \, , \tag{22}$$

$$\text{and for the insertion sources } \; \xi = (\gamma^a_\mu, \; \gamma, \; \gamma^a, \; \omega^a) \text{ and } \; \gamma_\tau = (\gamma^a_\mu, \; \gamma, \; \gamma^a) \, . \tag{23}$$

The quantization of the classical theory amounts to constructing a well-defined version of the formal functional integral respresentation for the generating functional W of the connected Green functions such that these functions satisfy the system of STI. Considering the formal expression for the modified generating functional

$$\exp \frac{1}{\hbar} W = \mathcal{N} \int [dA \, dh \, dB \, dc \, d\bar{c}] \exp\{-\frac{1}{\hbar} S_c\} \tag{24}$$

we observe that the quadratic part (10) appearing in S_c constitutes a well-defined Gaussian measure[6]. In a formal loop expansion of the remaining part of the exponent the emerging order $\hbar^0$, i.e. the tree approximation, is well-defined and satisfies

$$\mathcal{D} W|_{\hbar^0} = 0 \, , \tag{25}$$

which follows from (17) when using the invariance of the (formal) measure in (24) under BRS transformations. In the sequel we will inductively tackle all orders $\hbar^l$, $l \in \mathbb{N}$, of the loop expansion.

[6]Once we have introduced the regularization (30) the support of the measure consists of sufficiently well-behaved functions.

3 Flow Equations: Renormalizability without Slavnov-Taylor Identities

3.1 The Flow Equations for the SU(2) Yang-Mills Higgs model

The FE of Wilson's renormalization group is obtained as a differential equation w.r.t. the flow parameter Λ, which is the energy scale down to which the degrees of freedom have been integrated out, starting from the UV region. We will consider the generating functional of the connected amputated Green functions (CAG) which we denote as

$$L^{\Lambda,\Lambda_0}(\varphi_\tau, c, \bar{c}) \tag{26}$$

with the following explanations: We have introduced an UV regularization[7] Λ_0 to have a well-defined starting point, so that

$$0 \leq \Lambda \leq \Lambda_0 \, < \, \infty \; . \tag{27}$$

The functional $L^{\Lambda,\Lambda_0}(\varphi, c, \bar{c})$ is to be viewed as a formal power series in $\hbar$, since we are studying the perturbative renormalization problem in the loop expansion. To be more precise on its definition we write it as

$$L^{\Lambda,\Lambda_0} \, = \, \sum_{|n|=3}^{\infty} L^{\Lambda,\Lambda_0}_{l=0,n} \, + \, \sum_{l=1}^{\infty} \hbar^l \, \sum_{|n|=1}^{\infty} L^{\Lambda,\Lambda_0}_{l,n} \; . \tag{28}$$

Here the multiindex n denotes the number of field variables of each species appearing:

$$n = \, \{n_A, \, n_h, \, n_B, \, n_{\bar{c}}, \, n_c\}, \quad |n| := n_A + \, n_h + \, n_B + \, n_{\bar{c}} + \, n_c \; . \tag{29}$$

So for $|n| = 4$ we are e.g. regarding a four point function. (28) implies that, by definition, at 0 loop order L^{Λ,Λ_0} contains no contribution from the one- or two-point functions. With this restriction it is the generating functional of the CAG of the following theory:
i) The propagators are those from (12) including the regulating factor

$$\sigma_{\Lambda,\Lambda_0}(p^2) \, = \, \frac{\sigma_{\Lambda_0}(p^2) \, - \, \sigma_\Lambda(p^2)}{\sigma_{\Lambda_0}(0)} \quad \text{with} \quad \sigma_\Lambda(p^2) \, = \, e^{-\frac{1}{\Lambda^6}[(p^2+m^2)(p^2+\alpha m^2)(p^2+M^2)]} \; . \tag{30}$$

[7] Furthermore we should restrict the theory to a finite volume V as long as field independent vacuum terms are generated by the flow, which diverge in infinite volume by translation invariance. We do not make this explicit here and refer the interested reader to previous work [KKSa, KK3].

In the sequel this choice of the cutoff function turns out to be technically convenient[8]. Besides being explicit it permits to verify easily the following bounds on the regularized propagators $C^{\Lambda,\Lambda_0}(p) \; := \; C(p)\,\sigma_{\Lambda,\Lambda_0}(p^2)$

$$|(\prod_{i=1}^{|w|}\frac{\partial}{\partial p_{\mu_i}})\frac{\partial}{\partial\Lambda}C^{\Lambda,\Lambda_0}(p)| \leq \left\{ \begin{array}{ccc} C & , & \text{for} \quad 0\leq\Lambda\leq m \\ \Lambda^{-3-|w|}\,\mathcal{P}(|p|/\Lambda)\,\sigma_\Lambda(p^2) & , & \text{for} \quad m\leq\Lambda\leq\Lambda_0 \end{array} \right\} . \tag{31}$$

Here and in the following $\mathcal{P}$ denotes (each time it appears possibly a new) polynomial with nonnegative coefficients. These as well as the constant C depend on α, m, M, $|w|$, but not on p, Λ, Λ_0.
ii) The vertices are to be taken from our starting bare action (interaction Lagrangian including counter terms)

$$L^0 \; := \; L^{\Lambda_0,\Lambda_0}\,. \tag{32}$$

In the case of an invariant regularization we would choose here $S_{\rm BRS}$ from (9b), modified by including counter terms of any order $\hbar^l$, $l\geq 1$, of the same structure and by excluding the 0-loop quadratic part. In our case such a restricted choice would not allow to prove restoration of the STI. Therefore we will allow at first for all counter terms permitted by the unbroken global symmetries of the theory, i.e. O(4) and $\mathrm{SO}(3)_{\rm iso}$.

These terms will then become unique functions of the renormalization conditions chosen. There are 37 such local terms of dimension ≤ 4, corresponding to those listed in Appendix A. At the tree level $l=0$ we shall always consider the terms with $|n|+|w|\leq 4$ to be given by (11). We denote by

$$(2\pi)^{4(|n|-1)}\delta^n_{\Phi(p)}L_l^{\Lambda,\Lambda_0}|_{\Phi\equiv 0} \; = \; \delta(p_1+\ldots+p_{|n|})\,\mathcal{L}^{\Lambda,\Lambda_0}_{l,n}(p_1,\ldots,p_{|n|-1}) \tag{33}$$

the n-point CAG of loop order l involving the indicated number of (A_μ, h, B, $\bar{c}$, c) fields. We will also write $\vec{p}$ for (p_1, ..., $p_{|n|-1}$) in the following. We stay somewhat unprecise about the momentum assignment to the fields since this would unnecessarily blow up the notation. We also omit vector and isovector indices. Finally we will also use the shorthand

$$\partial^w := \prod_{i=1}^{|n|-1}\prod_{\mu=1}^{4}(\frac{\partial}{\partial p_{\mu_i}})^{w_{i,\mu}} \quad \text{with} \quad w=(w_{i,1},\ldots,w_{|n|-1,4}),\; |w|=\sum w_{i,\mu}\,. \tag{34}$$

[8] There is of course a lot of arbitrariness in this choice. What is needed is a sufficiently well-behaved function tending to 1 for $\Lambda\to 0$, $\Lambda_0\to\infty$, which is essentially supported for momenta between Λ and Λ_0. The verification of the restoration of the STI in Ch.4 would be somewhat easier using a suitable regulating function with compact support of the type $\sigma_\Lambda(p) = K(\frac{p^2+m^2}{\Lambda^2})$, where $K(x)=1, x\leq 1$, $K(x)=0, x\geq 2$, K monotonic and smooth. But the choice (30) allows to perform the analytic continuation to Minkowski space as shown in [KKSc], and it has the advantage that $(\sigma_\Lambda(p))^{-1}$ is well-defined. Avoiding its appearance is possible, but sometimes needs detours.

The Flow equations (FE) have been derived quite generally several times, so we tend to be short. The Wilson FE written for L^{Λ,Λ_0} takes the form[9]

$$e^{-\frac{1}{\hbar}(L^{\Lambda,\Lambda_0}+I^{\Lambda,\Lambda_0})} = e^{\hbar\Delta(\Lambda,\Lambda_0)}\, e^{-\frac{1}{\hbar}L^0}\ . \tag{35}$$

Here $\Delta(\Lambda,\Lambda_0)$ is the functional Laplace operator which in our theory takes the form

$$\Delta(\Lambda,\Lambda_0)=\frac{1}{2}\langle\frac{\delta}{\delta A^a_\mu},\, C^{\Lambda,\Lambda_0}_{\mu\nu}\frac{\delta}{\delta A^a_\nu}\rangle+\ \frac{1}{2}\langle\frac{\delta}{\delta h},\, C^{\Lambda,\Lambda_0}\frac{\delta}{\delta h}\rangle+\ \frac{1}{2}\langle\frac{\delta}{\delta B^a},\, S^{\Lambda,\Lambda_0}\frac{\delta}{\delta B^a}\rangle+\ \langle\frac{\delta}{\delta c^a},\, S^{\Lambda,\Lambda_0}\frac{\delta}{\delta \bar{c}^a}\rangle\ . \tag{36}$$

Using our shorthand notation we obtain the FE for the CAG $\mathcal{L}^{\Lambda,\Lambda_0}_{l,n}$ from (35) on deriving w.r.t. Λ, expanding L as in (28) and using (33)

$$\partial_\Lambda\partial^w\,\mathcal{L}^{\Lambda,\Lambda_0}_{l,n}(\vec{p}) \;=\sum_{n',|n'|=|n|+2} c_{n'}\int_k (\partial_\Lambda C^{\Lambda,\Lambda_0}(k))\,\partial^w\mathcal{L}^{\Lambda,\Lambda_0}_{l-1,n'}(\vec{p},k,-k) \tag{37}$$

$$-\sum_{\substack{l_1+l_2=l,\, w_1+w_2+w_3=w\\ n_1,n_2,|n_1|+|n_2|=|n|+2}}\left[c_{n_1,n_2}\,\partial^{w_1}\mathcal{L}^{\Lambda,\Lambda_0}_{l_1,n_1}(p_1,\ldots,p_{|n_1|-1})(\partial^{w_3}\partial_\Lambda C^{\Lambda,\Lambda_0}(p'))\ \partial^{w_2}\mathcal{L}^{\Lambda,\Lambda_0}_{l_2,n_2}(-p',\ldots,p_{|n|-1})\right]_{s,a}.$$

The constants $c_{n'}$, c_{n_1,n_2} are combinatorial. The field assignment of the propagators C^{Λ,Λ_0} is not written, it is implicit in the multiindices n', n_1, n_2 related to n. On the r.h.s. the integrated momentum k refers to that of the fields from $n'-n$, and $-p'=p_1+\ldots+p_{|n_1|-1}$. Furthermore the subscripts s,a indicate (anti)symmetrization according to the statistics of the various fields, since we assume the $\mathcal{L}^{\Lambda,\Lambda_0}_{l,n}$ to be (anti)symmetrized from the beginning.

3.2 Renormalizability

The system of differential FE (37) can be integrated inductively, using mixed boundary conditions (b.c.) :

A_1) At $\Lambda=\Lambda_0$ the n point functions with $|n|+|w|>4$, i.e. the irrelevant ones, are supposed to be smooth functions of $\vec{p}$, Λ_0 obeying the bounds

$$|\partial^w\mathcal{L}^{\Lambda_0,\Lambda_0}_{l,n}(\vec{p})|\,\le\,\Lambda_0^{4-|n|-|w|}\,\mathcal{P}_1(log\frac{\Lambda_0}{m})\,\mathcal{P}_2(\frac{|\vec{p}|}{\Lambda_0})\ ,\quad |n|+|w|\,\ge\,5\ . \tag{38}$$

The standard case are b.c., where the r.h.s. of (38) vanishes. We need to be slightly more general to compensate for effects of the cutoff function σ_{0,Λ_0}, see Ch.4, (107, 108).

A_2) At $\Lambda=0$ the CAG with $|n|+|w|\le 4$, i.e. the relevant ones, are fixed, order by order in $\hbar$ at the renormalization point, which we choose at $\vec{p}=0$ for simplicity. The

[9] I^{Λ,Λ_0} is the vacuum functional which strictly speaking exists only in finite volume. Since it plays hardly any role in the following, we do not discuss this issue here and refer to [KKSa, KK3] for further comments.

renormalization conditions (r.c.) may be chosen weakly Λ_0-dependent, we restrict to smooth uniformly bounded functions of Λ_0 converging for $\Lambda_0 \to \infty$. Of course we always restrict to b.c. respecting the global (Euclidean and Iso-)symmetries.
With the FE we can inductively obtain the following bounds on the CAG $\mathcal{L}_{l,n}^{\Lambda,\Lambda_0}$:

Proposition 1 :

$$|\partial^w \mathcal{L}_{l,n}^{\Lambda,\Lambda_0}(\vec{p})| \leq (\Lambda+m)^{4-|n|-|w|}\, \mathcal{P}_1(log\frac{\Lambda+m}{m})\, \mathcal{P}_2(\frac{|\vec{p}|}{\Lambda+m})\,. \tag{39}$$

The polynomials $\mathcal{P}_1,\ \mathcal{P}_2$ have nonnegative coefficients depending on $l,\ n,\ w,\ \alpha,\ m,\ M$, but not on $\vec{p},\ \Lambda,\ \Lambda_0$.

We do not present a proof of the proposition since the line of thought is the same as in the references [KKSa, KK3, Kop] and restrict to few comments. It proceeds by induction upwards in the number of loops and for given loop order upwards in $|n|$ (in contrast to the procedure employed when expanding in a coupling constant : There one proceeds downwards in $|n|$. For given l, n we proceed downwards in $|w|$, starting from some arbitrary[10] $|w_{max}| \geq 3$. Thus we have to start at loop order $l = 0$ and from $|n| = 3$, since $L_{l=0}^{\Lambda,\Lambda_0}$ does not contain contributions for $|n| \leq 2$. (37) immediately gives

$$\mathcal{L}_{0,n}^{\Lambda,\Lambda_0}(\vec{p}) \;=\; \mathcal{L}_{0,n}^{\Lambda_0,\Lambda_0}(\vec{p})\,, \quad |n| = 3\,,$$

since the r.h.s. vanishes. Thus the bound is satisfied. For $|n| = 4,\ l = 0$ we may also fix the b.c. at $\Lambda = \Lambda_0$, *if we want to read them off the action (11)*, since here the second term on the r.h.s. of (37) contributes and leads to a one particle reducible difference between $\mathcal{L}_{0,n}^{\Lambda,\Lambda_0}$ and $\mathcal{L}_{0,n}^{\Lambda_0,\Lambda_0}$. This digression of the rules A_1), A_2) is a pure matter of convenience however. The inductive proof then proceeds by inserting the induction hypothesis on the r.h.s. of the FE (which has already been bounded) and performing the momentum and Λ-integrals, starting from the respective b.c. and using the bound (31). An important point to note is the following : Which bounds for the $\mathcal{L}^{\Lambda,\Lambda_0}$ can be obtained, depends only on the b.c. imposed and on the propagators (and dimensionality). Note finally that for the purpose of renormalizability only the bound on $\mathcal{L}^{0,\Lambda_0}$ in the limit $\Lambda_0 \to \infty$ is needed. The rest is of technical nature. In the next chapter we want to make use of the following also somewhat technical

Corollary : For given $l_0 > 0$ and $n_0,\ w_0$ with $|n_0| + |w_0| \leq 4$ we assume that the b.c. on

[10] The minimal value of 3 is needed, because for the relevant terms the passage from the fixed momentum, at which the renormalization conditions are imposed, to any momentum is achieved by the Schlömilch or integrated Taylor formula [KKSa,Pol]. For the two point function there thus appear up to three derivatives. If one also wants to prove smoothness one has to admit for arbitrarily high $|w_{max}|$.

the CAG $\partial^w \mathcal{L}_{l,n}^{\Lambda,\Lambda_0}$, ($|w| \le |w_{max}|$), have been imposed in agreement with A_1), A_2) for $l < l_0$ and arbitrary $n,\ w$; and for $l = l_0$ and $|n| < |n_0|$ and $|w| \le |w_0|$. Suppose that we fix the b.c. for $\partial^{w_0} \mathcal{L}_{l_0,n_0}^{\Lambda,\Lambda_0}$ 'on the wrong side', i.e. at Λ_0, such that it obeys the bound

$$|\partial^{w_0} \mathcal{L}_{l_0,n_0}^{\Lambda_0,\Lambda_0}(0)| \le \Lambda_0^{4-|n_0|-|w_0|}\, \mathcal{P}(log(\Lambda_0/m))\,. \tag{40}$$

Then we also have

$$|\partial^{w_0} \mathcal{L}_{l_0,n_0}^{\Lambda,\Lambda_0}(0)| \le \Lambda_0^{4-|n_0|-|w_0|}\, \mathcal{P}(log(\Lambda_0/m))\,. \tag{41}$$

Proof : Due to our assumptions, the r.h.s. of the FE (37) is bounded by (39), since the bounds on all terms preceding (l_0, n_0, w_0) in the induction remain unchanged apart from those with $|w| > |w_0|$. Those are not needed however because we only make a statement at the renormalization point $\vec{p} = 0$ and thus do not require a bound on the Taylor remainder. The deterioration of the bound then stems from both the b.c. contribution (40) *and* from the fact that the r.h.s. of the FE has to be integrated from Λ_0 to Λ (instead of integrating from 0 to Λ), i.e. from the wrong side. This gives the bound

$$|\partial^{w_0} \mathcal{L}_{l_0,n_0}^{\Lambda,\Lambda_0}(0)| \le \Lambda_0^{4-|n_0|-|w_0|}\, \mathcal{P}_1(log(\Lambda_0/m)) \,+\, |\int_{\Lambda}^{\Lambda_0} d\Lambda'\, \Lambda'^{4-|n_0|-|w_0|-1}\, \mathcal{P}_2(log(\Lambda'/m))|$$

$$\le \Lambda_0^{4-|n_0|-|w_0|}\, \mathcal{P}_3(log(\Lambda_0/m))\,.$$

Note that the bound does not improve, if we set the b.c. for $\partial^{w_0} \mathcal{L}_{l_0,n_0}^{\Lambda_0,\Lambda_0}(0)$ equal to zero. ■

We remark that statements similar to that of the Corollary could also be extended to general external momenta, they are not needed however. In response to the remarks made before one may ask oneself whether the previous bounds (39) may be improved, if the b.c. are in some sense smaller. This is indeed the case. Regard e.g. the CAG containing an odd number of scalar fields, i.e. $n_h + n_B \in 2\mathbb{N} - 1$. Then the following improved bounds hold :

$$|\partial^w \mathcal{L}_{l,n}^{\Lambda,\Lambda_0}(\vec{p})| \le\ (\Lambda + m)^{3-|n|-|w|}\, \mathcal{P}_1(log\frac{\Lambda + m}{m})\, \mathcal{P}_2(\frac{|\vec{p}|}{\Lambda + m})\,. \tag{42}$$

The main reason why we may expect an improvement of power counting for those terms in our theory is that, as can be seen in App.A , at $l = 0$ the terms in question are all proportional to a mass factor. Since we will not need such sharpened statements we do not give a proof of (42) here. As usual the bound on the Green functions should be complemented by a convergence statement, since (39, 42) would still admit bounded but oscillating solutions[11]. Convergence follows from

[11] a possibility generally only envisaged by mathematical physicists since such oscillations are counterintuitive to any experience from calculations

Proposition 2 :

$$|\partial_{\Lambda_0}\partial^w \mathcal{L}^{\Lambda,\Lambda_0}_{l,n}(\vec{p})| \le \frac{1}{\Lambda_0^2}(\Lambda+m)^{5-|n|-|w|}\,\mathcal{P}_1(log(\Lambda_0/m))\,\mathcal{P}_2(\frac{|\vec{p}|}{\Lambda+m})\,. \tag{43}$$

As before the nonnegative coefficients in the (new) polynomials $\mathcal{P}_i$ may depend on $l,\ n,\ w,\ \alpha,\ m,\ M$, but not on $\vec{p},\ \Lambda,\ \Lambda_0$. For the proof, which follows the same inductive scheme, we refer again to the earlier references [KKSa, KK3, Kop].

3.3 Bounds on Green functions with Operator Insertions

The problem of renormalizing Green functions with operator insertions has been studied quite generally in [KK3, KK4]. Again we state the propositions needed for SU(2) Yang-Mills theory without proofs, restricting to remarks on the (minor) modifications needed. We have to deal with two kinds of operator insertions here. The first are the BRS insertions (13a)-(13d). These are defined as operator insertions of dimension 2, ghost number one for (13a)-(13c) and ghost number 2 for (13d), which transform as vector-isovector, scalar-isoscalar, scalar-isovector and scalar-isovector respectively. By the general renormalization theory we thus have to allow for all counter terms of dimension ≤ 2 and of the same symmetry properties. In the bare action the insertions take the form

$$\psi^a_\mu(x) = R^0_1\,\partial_\mu c^a(x) + R^0_2\,g\,\epsilon^{arb}A^r_\mu(x)\,c^b(x), \tag{44a}$$

$$\psi(x) = -R^0_3\,\frac{1}{2}g\,B^a(x)c^a(x), \tag{44b}$$

$$\psi^a(x) = R^0_4\,m\,c^a(x) + R^0_5\,\frac{1}{2}g\,h(x)\,c^a(x) + R^0_6\,\frac{1}{2}g\,\epsilon^{arb}B^r(x)\,c^b(x), \tag{44c}$$

$$\Omega^a(x) = R^0_7\,\frac{1}{2}g\,\epsilon^{apq}c^p(x)c^q(x)\,, \tag{44d}$$

where we demand

$$R^0_i \;=\; 1\,+\,O(\hbar)\ , \tag{45}$$

i.e. the counter terms are again viewed as formal power series in $\hbar$, and we of course assume the insertions to agree with (13a-13d) at the tree level.

The following remark might be helpful, as regards the transformation (14e) of the anti-ghost : We do not introduce constants $R^0_8,\ \ldots,\ R^0_{11}$, corresponding to the terms of dimension ≤ 2 with the same symmetry properties (besides the ones in (14e) these are $h\,B^a$ and $\varepsilon^{abc}c^b\bar{c}^c$). The claim implicit (not only here, but throughout the literature) and verified in Ch.4 is then that it is possible to obtain a finite renormalized theory[12] satisfying the STI, by fixing these

[12] This is related to the fact that (14e) is linear in Φ.

constants at $\Lambda = \Lambda_0$, i.e. on the wrong side ; in fact setting $R_8^0,\, R_9^0 \;=\, 1,\; R_{10}^0,\, R_{11}^0 \;=\; 0$. In the more general case one would have to admit arbitrary values for these four constants and to introduce another source for the respective composite operator. The (violated) STI (see below (75, 82, 98)) would then take a more symmetric form, the terms involving A^a_μ, B^a being replaced by another one of the form $\langle c^a,\, D\, L_{\overline{\gamma}^a}\rangle$.

The insertions may be generated by the respective sources as in (16), we set

$$L_\xi^{\Lambda_0,\Lambda_0} \;=\; \int dx\, \{\gamma^a_\mu(x)\psi^a_\mu(x) + \gamma(x)\psi(x) + \gamma^a(x)\psi^a(x) + \omega^a(x)\Omega^a(x)\}\,, \tag{46}$$

and also

$$\tilde L^{\Lambda_0,\Lambda_0} \;=\; L^{\Lambda_0,\Lambda_0} \;+\; L_\xi^{\Lambda_0,\Lambda_0}\;. \tag{47}$$

We again get a Wilson FE (cf. (35)) for $\tilde L^{\Lambda,\Lambda_0}$ generating the CAG with operator insertions[13]

$$e^{-\frac{1}{\hbar}(\tilde L^{\Lambda,\Lambda_0}+\tilde I^{\Lambda,\Lambda_0})} \;=\; e^{\hbar\Delta(\Lambda,\Lambda_0)}\; e^{-\frac{1}{\hbar}\tilde L^{\Lambda_0,\Lambda_0}}\;. \tag{48}$$

Restricting our attention to CAG with *one* insertion, e.g.

$$L^{\Lambda,\Lambda_0}_{\gamma(x)} := \frac{\delta \tilde L^{\Lambda,\Lambda_0}}{\delta\gamma(x)}|_{\xi=0} \tag{49}$$

(similarly for the other insertions) we obtain by deriving (48) w.r.t. Λ a *linear* FE for $L^{\Lambda,\Lambda_0}_{\gamma(x)}$. Writing similarly as in(33)

$$(2\pi)^{4(|n|-1)}\delta^n_{\Phi(p)} L^{\Lambda,\Lambda_0}_{\gamma(q);l}|_{\Phi\equiv 0} \;=\; \delta(q+p_1+\ldots+p_{|n|})\, \mathcal{L}^{\Lambda,\Lambda_0}_{\gamma(q);l,n}(p_1,\ldots,p_{|n|-1}) \tag{50}$$

we obtain the differential FE for CAG with one insertion

$$\partial_\Lambda \partial^w\, \mathcal{L}^{\Lambda,\Lambda_0}_{\gamma(q);l,n}(\vec p) \;=\; \sum_{n',|n'|=|n|+2} c_{n'} \int_k (\partial_\Lambda C^{\Lambda,\Lambda_0}(k))\, \partial^w \mathcal{L}^{\Lambda,\Lambda_0}_{\gamma(q);l-1,n'}(\vec p,k,-k) \tag{51}$$

$$-\sum_{\substack{l_1+l_2=l,\; w_1+w_2+w_3=w\\ n_1,n_2,|n_1|+|n_2|=|n|+2}} \Big[c_{n_1,n_2}\, \partial^{w_1}\mathcal{L}^{\Lambda,\Lambda_0}_{\gamma(q);l_1,n_1}(p_1,\ldots,p_{|n_1|-1})(\partial^{w_3}\partial_\Lambda C^{\Lambda,\Lambda_0}(p'))\, \partial^{w_2}\mathcal{L}^{\Lambda,\Lambda_0}_{l_2,n_2}(-p',\ldots,p_{|n|-1})\Big]_{s,a}$$

the notation being that of (37). Since ghost and antighost in (36) do not appear symmetrically, the $\bar c$ (c)-derivative appears once in n_1 (n_2) and once in n_2 (n_1). In the following we denote for shortness by $\xi(q)$ any of the sources $\gamma^a_\mu(q)$, $\gamma(q)$, $\gamma^a(q)$, $\omega^a(q)$. Obviously each of the insertions leads to a FE as (51). In the derivation of (51) no use is made of the specific

[13] We will only regard insertions with nonvanishing ghost number. Therefore the vacuum functional $\tilde I$ equals I, since there are no vacuum diagrams with nonvanishing ghost number, due to ghost number conservation under the flow. Thus we will always write I subsequently.

kind of insertion considered. Thus even more generally we replace $\xi(q)$ by $\chi(q)$ when talking of an insertion of dimension D (instead of 2). This is because we also want to cover the CAG with one insertion of dimension 5 describing the BRS violating terms of the regularized theory. This insertion is analysed in Ch.4.1. The particular kind of insertion chosen only comes into play when considering the b.c., which are fixed as follows :

B_1) At $\Lambda = \Lambda_0$ the n point functions $\partial^w \mathcal{L}^{\Lambda,\Lambda_0}_{\chi(q);l,n}$ with $|n|+|w|>D$, i.e. the irrelevant ones, are supposed to obey the bounds (cf. A_1, (38))

$$|\partial^w \mathcal{L}^{\Lambda_0,\Lambda_0}_{\chi(q);l,n}(\vec{p})| \leq \Lambda_0^{D-|n|-|w|}\,\mathcal{P}_1(log\frac{\Lambda_0}{m})\,\mathcal{P}_2(\frac{|\vec{p}|}{\Lambda_0})\,, \quad |n|+|w| > D\,. \tag{52}$$

B_2) At $\Lambda = 0$ the CAG with $|n|+|w| \leq D$, i.e. the relevant ones, are fixed, order by order in $\hbar$ at the renormalization point $\vec{p}=0$, with the same restrictions as in A_2).

Again (51) lends itself to an inductive scheme through which we may prove the renormalizability of the CAG with insertion. For the $\mathcal{L}^{\Lambda,\Lambda_0}_{\xi(q);l,n}$ there are seven free r.c. which fix the seven parameters R_i^0 from (45). For the CAG $\mathcal{L}^{\Lambda,\Lambda_0}_{\chi(q);l,n}$ with insertion $L_1^{\Lambda_0,\Lambda_0}$ from (67) we have to fix 53 r.c. corresponding to the list in App.C. Under these conditions our inductive scheme may now also be employed to prove boundedness and convergence of inserted Green functions.

Proposition 3:

$$|\partial^w \mathcal{L}^{\Lambda,\Lambda_0}_{\chi(q);l,n}(\vec{p})| \leq (\Lambda+m)^{D-|n|-|w|}\,\mathcal{P}_1(log\frac{\Lambda+m}{m})\,\mathcal{P}_2(\frac{|\vec{p}|}{\Lambda+m})\,, \tag{53}$$

$$|\partial_{\Lambda_0}\partial^w \mathcal{L}^{\Lambda,\Lambda_0}_{\chi(q);l,n}(\vec{p})| \leq \frac{(\Lambda+m)^{D+1-|n|-|w|}}{\Lambda_0^2}\,\mathcal{P}_1(log(\Lambda_0/m))\,\mathcal{P}_2(\frac{|\vec{p}|}{\Lambda+m})\,. \tag{54}$$

Whereas the bounds from Proposition 3 are sufficient for our purposes as regards the functions $\mathcal{L}^{\Lambda,\Lambda_0}_{\xi(q);l,n}$, we need a stronger result for the BRS violating insertions $\mathcal{L}^{\Lambda,\Lambda_0}_{\chi(q);l,n}$, which we can achieve on imposing further restrictions on the b.c. It is important in this respect that the FE for the inserted CAG is linear. This implies e.g. that multiplying all CAG with a Λ- independent factor gives a new solution. If we want to show that the CAG $\mathcal{L}^{\Lambda,\Lambda_0}_{\chi(q);l,n}$ from Ch.4.1 vanish in the limit $\Lambda_0 \to \infty$, the strategy is thus to reveal a negative power of Λ_0, which can be factorized from the CAG $\mathcal{L}^{\Lambda,\Lambda_0}_{\chi(q);l,n}$. It is quite conceivably a sufficient condition for achieving this, to require that all r.c. be bounded by a negative power of Λ_0. The main issue of Ch.4 will be to prove that there exist r.c. on the CAG such that the inserted CAG describing BRS violation obey such suppressed r.c. Once this is accomplished we can rely on the following proposition for the restoration of BRS invariance :

Proposition 4: Replace the statements from B_2) on the renormalization conditions by

B_3) At $\Lambda=0$ the $\mathcal{L}^{0,\Lambda_0}_{\chi(q);l,n}$ with $|n|+|w|\leq D$ are fixed at order $\hbar^l$ and $\vec{p}=0$ to be smooth functions of Λ_0 bounded by

$$\frac{1}{\Lambda_0}\,\mathcal{P}(log(\Lambda_0/m))\ . \tag{55}$$

Then we have the bound

$$|\partial^w \mathcal{L}^{\Lambda,\Lambda_0}_{\chi(q);l,n}(\vec{p})| \leq \frac{1}{\Lambda_0}\,(\Lambda+m)^{D+1-|n|-|w|}\,\mathcal{P}_1(log(\Lambda_0/m))\,\mathcal{P}_2(\frac{|\vec{p}|}{\Lambda+m})\ . \tag{56}$$

Again we do not give a proof, but refer to our previous remarks, to [KK3] and in particular to Prop.7 in the paper on QED [KK2], where similar results were obtained in the more complicated situation of a massless theory. Proposition 4 obviously shows that the CAG $\mathcal{L}^{\Lambda,\Lambda_0}_{\chi(q);l,n}$ vanish for $\Lambda_0\to\infty$. We remark that in Ch.4 we will arrange for r.c. such that the bound (55) can be set to 0. This does not improve (56), because of the nonvanishing b.c. for the irrelevant terms (see B1), (52) above).

4 Restoration of the Slavnov-Taylor Identities

4.1 Violated Slavnov-Taylor Identities for Connected and Proper Green functions

Once the physical free parameters of the theory, i.e. g, λ, m and the gauge fixing parameter α [14] have been fixed, the Yang-Mills-Higgs theory should be uniquely determined up to normalizations of the fields. The standard tool to enforce this uniqueness are the Slavnov-Taylor-identities. Whereas their role is twofold in renormalization procedures based on invariant regularization schemes - apart from assuring uniqueness and physical gauge invariance, they also serve as a technical tool to show inductively that the theory can be renormalized without introducing counter terms not present in the bare action - we only have to ensure their validity for the first purpose. At an intermediate stage they are inevitably violated by the regularization in momentum space, as gauge invariance is. We want to show that they hold after removing the regularization, if we choose the renormalization conditions properly. Our starting point is the generating functional of the regularized Green functions at the physical value $\Lambda=0$ of the flow parameter. Remembering (21,22) we write

$$\langle\Phi,\,K\rangle\ =\ \int dx\,\{\sum_\tau \varphi_\tau(x)J_\tau(x)\ +\ \bar{c}^a(x)\eta^a(x)\ +\ \bar{\eta}^a(x)c^a(x)\}\ . \tag{57}$$

[14] on which physical quantities should not depend

The Gaussian measure $d\mu_{\Lambda_0}(\Phi)$ corresponding to the quadratic form $\frac{1}{\hbar}\, Q^{\Lambda_0}$ with

$$Q^{\Lambda_0} = \frac{1}{2}\langle A^a_\mu,\, (C^{0,\Lambda_0})^{-1}_{\mu\nu}\, A^a_\nu\,\rangle + \frac{1}{2}\langle h,\, (C^{0,\Lambda_0})^{-1} h\,\rangle + \frac{1}{2}\langle B^a,\, (S^{0,\Lambda_0})^{-1} B^a\,\rangle - \langle \bar{c}^a,\, (S^{0,\Lambda_0})^{-1} c^a\,\rangle \tag{58}$$

is given by its characteristic functional

$$\int d\mu_{\Lambda_0}(\Phi)\, e^{\frac{1}{\hbar}\langle \Phi,\, K\rangle} \;=\; e^{\frac{1}{\hbar}P(K)} \tag{59}$$

with

$$P(K) \;=\; \frac{1}{2}\langle j^a_\mu,\, C^{0,\Lambda_0}_{\mu\nu}\, j^a_\nu\rangle \;+\; \frac{1}{2}\langle s,\, C^{0,\Lambda_0}\, s\rangle \;+\; \frac{1}{2}\langle b^a,\, S^{0,\Lambda_0} b^a\rangle \;-\; \langle \overline{\eta}^a,\, S^{0,\Lambda_0}\eta^a\rangle\,. \tag{60}$$

The generating functional of the regularized Green functions may now be written as

$$Z^{0,\Lambda_0}(K) \;=\; \int d\mu_{\Lambda_0}(\Phi)\, e^{-\frac{1}{\hbar}L^{\Lambda_0,\Lambda_0} + \frac{1}{\hbar}\langle \Phi,\, K\rangle}\;. \tag{61}$$

Defining *regularized* BRS variations of the fields through

$$\delta_{BRS}\,\varphi_\tau(x) \;=\; -(\sigma_{0,\Lambda_0}\psi_\tau)(x)\,\varepsilon\,,\quad \delta_{BRS}\,c^a(x) \;=\; -(\sigma_{0,\Lambda_0}\Omega^a)(x)\,\varepsilon\,, \tag{62}$$

$$\delta_{BRS}\,\bar{c}^a(x) \;=\; -[\sigma_{0,\Lambda_0}(\frac{1}{\alpha}\partial_\nu A^a_\nu - m\,B^a)](x)\,\varepsilon\;,$$

the BRS transform of the Gaussian measure is given by

$$d\mu_{\Lambda_0}(\Phi) \;\mapsto\; d\mu_{\Lambda_0}(\Phi)\Big\{1 \;+\; \frac{1}{\hbar}\sum_\tau\langle \varphi_\tau,\, (C^{0,\Lambda_0}_\tau)^{-1}\sigma_{0,\Lambda_0}\psi_\tau\rangle\,\varepsilon \;-\; \frac{1}{\hbar}\langle \bar{c}^a,\, (S^{0,\Lambda_0})^{-1}\sigma_{0,\Lambda_0}\Omega^a\rangle\,\varepsilon \tag{63}$$

$$+\,\frac{1}{\hbar}\langle\frac{1}{\alpha}\partial_\nu A^a_\nu - m\,B^a,\, \sigma_{0,\Lambda_0}(S^{0,\Lambda_0})^{-1}c^a\rangle\,\varepsilon\Big\} \;=\; d\mu_{\Lambda_0}(\Phi)\Big\{1 \;-\; \frac{1}{\hbar}\,\delta_{BRS}\,Q^{\Lambda_0}\Big\}\;.$$

The BRS-variation of the measure has mass dimension 5, since σ_{0,Λ_0} just cancels its inverse appearing in the inverted propagators in (63). This is convenient, and it is the basic reason why we regularized the BRS-transformation. Requiring the invariance of the functional integral in (61) under (regularized) BRS-transformations of the field variables [15], (62) provides us with the Violated Slavnov-Taylor identities (VSTI) :

$$0 \;\stackrel{!}{=}\; \int d\mu_{\Lambda_0}(\Phi)\, e^{-\frac{1}{\hbar}L^{\Lambda_0,\Lambda_0} + \frac{1}{\hbar}\langle \Phi,\, K\rangle}\,\{\delta_{BRS}\,\langle \Phi,\, K\rangle \;-\; \delta_{BRS}\,(Q^{\Lambda_0} \;+\; L^{\Lambda_0,\Lambda_0})\}\;. \tag{64}$$

[15]These transformations of variables and consequently (64) can be given rigorous meaning for the regularized Gaussian integrals. Arguing formally (64) amounts to the somewhat sloppy statement that the Jacobian of the BRS-transformation equals 1 which in turn has rigorous meaning for the lattice regularization, see e.g. [Rei].

The BRS variations in (64) can be generated using an appropriate operator insertion:
i) First we define the modified generating functional using (47)

$$\tilde{Z}^{0,\Lambda_0}(K,\xi) \;=\; \int d\mu_{\Lambda_0}(\Phi)\, e^{-\frac{1}{\hbar}\tilde{L}^{\Lambda_0,\Lambda_0}+\frac{1}{\hbar}\langle\Phi,\,K\rangle} \tag{65}$$

together with the regularized BRS operator (compare to (18))

$$\mathcal{D}_{\Lambda_0}=\sum_{\tau}\langle J_\tau,\,\sigma_{0,\Lambda_0}\frac{\delta}{\delta\gamma_\tau}\rangle+\langle\bar{\eta}^a,\,\sigma_{0,\Lambda_0}\frac{\delta}{\delta\omega^a}\rangle+\langle(\frac{1}{\alpha}\partial_\nu\frac{\delta}{\delta j^a_\nu}-m\frac{\delta}{\delta b^a}),\,\sigma_{0,\Lambda_0}\eta^a\rangle\;. \tag{66}$$

ii) Secondly we define the terms emerging from the BRS-noninvariance of the action to form the insertion $L_1^{\Lambda_0,\Lambda_0}$ with ghost number 1

$$L_1^{\Lambda_0,\Lambda_0}\,\varepsilon:=-\delta_{BRS}(Q^{\Lambda_0}\;+\;L^{\Lambda_0,\Lambda_0})\;. \tag{67}$$

Due to the regularizing factor σ_{0,Λ_0} in (62) the insertion $L_1^{\Lambda_0,\Lambda_0}$ is not a local operator. Using (67) we introduce the generating functional

$$Z_\chi^{0,\Lambda_0}(K):=\int d\mu_{\Lambda_0}(\Phi)\, e^{-\frac{1}{\hbar}(L^{\Lambda_0,\Lambda_0}+\chi\,L_1^{\Lambda_0,\Lambda_0})+\frac{1}{\hbar}\langle\Phi,\,K\rangle} \tag{68}$$

for $\chi\in\mathbb{R}$. Now the VSTI (64) can be rewritten as

$$\mathcal{D}_{\Lambda_0}\,\tilde{Z}^{0,\Lambda_0}(K,\xi)|_{\xi\equiv 0}\;=\;\frac{d}{d\chi}Z_\chi^{0,\Lambda_0}(K)|_{\chi=0}\;. \tag{69}$$

The modified functionals from (65, 68) permit to define the generating functionals of the corresponding CAG with the respective insertions

$$\tilde{Z}^{0,\Lambda_0}(K,\xi)\;=\;e^{\frac{1}{\hbar}P(K)}\,e^{-\frac{1}{\hbar}(I^{0,\Lambda_0}+\tilde{L}^{0,\Lambda_0}(\varphi_\tau,c,\bar{c};\xi))}, \tag{70}$$

$$Z_\chi^{0,\Lambda_0}(K)\;=\;e^{\frac{1}{\hbar}P(K)}\,e^{-\frac{1}{\hbar}(I^{0,\Lambda_0}+L_\chi^{0,\Lambda_0}(\varphi_\tau,c,\bar{c}))}, \tag{71}$$

with the relations

$$\varphi_\tau(x)=\int dy\, C_\tau^{0,\Lambda_0}(x-y)J_\tau(y),\; c^a(x)=-\int dy\, S^{0,\Lambda_0}(x-y)\eta^a(y),\; \bar{c}^a(x)=-\int dy\, S^{0,\Lambda_0}(x-y)\overline{\eta}^a(y) \tag{72}$$

between the variables of the Z and L functionals. Introducing the shorthand

$$D_\tau\;=\;\Big((-\Delta+m^2)\delta_{\mu\nu}-\frac{1-\alpha}{\alpha}\partial_\mu\partial_\nu\,,\;-\Delta+M^2\,,\;-\Delta+\alpha m^2\equiv D\Big) \tag{73}$$

for the inverted nonregularized propagators and also (remember (49))

$$L_1:=L_1^{0,\Lambda_0}\;=\;\frac{d}{d\chi}L_\chi^{0,\Lambda_0}|_{\chi=0}\,,\quad L:=L^{0,\Lambda_0}\;=\;\tilde{L}^{0,\Lambda_0}|_{\xi\equiv 0}\;(=\;L_\chi^{0,\Lambda_0}|_{\chi=0})\;, \tag{74}$$

since we will mostly regard the theory with Λ set to 0 in this section, we obtain from (69) via (70, 71, 72) the VSTI for the connected amputated functions CAG

$$L_1 \;=\; \langle c^a,\, D(\frac{1}{\alpha}\partial_\nu A_\nu^a - m\, B^a)\rangle - \langle c^a,\, \sigma_{0,\Lambda_0}(\partial_\nu \frac{\delta L}{\delta A_\nu^a} - m\frac{\delta L}{\delta B^a})\rangle + \sum_\tau \langle \varphi_\tau,\, D_\tau L_{\gamma_\tau}\rangle - \langle \bar{c}^a,\, D\, L_{\omega_a}\rangle\,. \tag{75}$$

Since we also have to regard the proper vertex functions we define in an intermediate step the generating functional of connected nonamputated Green functions[16]

$$e^{\frac{1}{\hbar}\tilde{W}(K,\xi)} \;=\; \frac{\tilde{Z}(K,\xi)}{\tilde{Z}(0,0)} \tag{76}$$

(leaving out again the upper indices $0,\Lambda_0$). From this we derive using (69, 71, 72)

$$\mathcal{D}_{\Lambda_0}\, \tilde{W}(K,\xi)|_{\xi=0} \;=\; -L_1(\varphi_\tau, c, \bar{c})\,. \tag{77}$$

The Legendre transform of $\tilde{W}$ now leads us to the generating functional of the proper vertex functions. We set

$$\tilde{\Gamma}(\underline{\varphi}_\tau, \underline{\bar{c}}, \underline{c}; \xi) \;+\; \tilde{W}(J_\tau, \eta, \overline{\eta}; \xi) \;=\; \int dy\, \{\sum_\tau \underline{\varphi}_\tau J_\tau \;+\; \underline{\bar{c}}\,\eta \;+\; \overline{\eta}\, \underline{c}\} \tag{78}$$

with the relations

$$J_\tau(x) \;=\; \frac{\delta\tilde{\Gamma}}{\delta\underline{\varphi}_\tau(x)}\,,\;\; \underline{\varphi}_\tau(x) \;=\; \frac{\delta\tilde{W}}{\delta J_\tau(x)}\,, \tag{79}$$

$$\eta^a(x) \;=\; \frac{\delta\tilde{\Gamma}}{\delta\underline{\bar{c}}^a(x)}\,,\;\; \underline{\bar{c}}^a(x) \;=\; -\frac{\delta\tilde{W}}{\delta\eta^a(x)}\,, \qquad \overline{\eta}^a(x) \;=\; -\frac{\delta\tilde{\Gamma}}{\delta\underline{c}^a(x)}\,,\;\; \underline{c}^a(x) \;=\; \frac{\delta\tilde{W}}{\delta\overline{\eta}^a(x)}\,.$$

Note that (78) says that $J_\tau, \ldots$ may be viewed as a formal power series in $\hbar$ with coefficients depending on the classical fields $\underline{\varphi}_\tau, \ldots$ These series may be inverted to express $\underline{\varphi}_\tau, \ldots$ as series in terms of $J_\tau, \ldots$ As a consequence of (78) the relations

$$\frac{\delta\tilde{\Gamma}}{\delta\gamma_\tau} \;+\; \frac{\delta\tilde{W}}{\delta\gamma_\tau} \;=\; 0 \tag{80}$$

and an analogous one for the derivative w.r.t. the source ω^a hold. Similarly as before we write

$$\Gamma \;=\; \tilde{\Gamma}|_{\xi\equiv 0}\,,\quad \Gamma_{\gamma_\tau(x)} \;=\; \frac{\delta\tilde{\Gamma}}{\delta\gamma_\tau(x)}|_{\xi\equiv 0}\,. \tag{81}$$

Then the VSTI for the proper vertex functions emerging from (77) (where the upper indices $\Lambda=0,\ \Lambda_0$ in (82,83,84) are understood) read

$$\sum_\tau \langle \frac{\delta\Gamma}{\delta\underline{\varphi}_\tau},\, \sigma_{0,\Lambda_0}\Gamma_{\gamma_\tau}\rangle \;-\; \langle \frac{\delta\Gamma}{\delta\underline{c}^a},\, \sigma_{0,\Lambda_0}\Gamma_{\omega^a}\rangle \;-\; \langle (\frac{1}{\alpha}\partial_\nu \underline{A}_\nu^a \;-\; m\,\underline{B}^a),\, \sigma_{0,\Lambda_0}\frac{\delta\Gamma}{\delta\underline{\bar{c}}^a}\rangle \;=\; \Gamma_1(\underline{\varphi}, \underline{\bar{c}}, \underline{c}) \tag{82}$$

[16]noting again that vacuum functionals should only appear before taking the infinite volume limit

with

$$\Gamma_1(\underline{\varphi}, \underline{\bar{c}}, \underline{c}) \;=\; L_1(\varphi, \bar{c}, c) \tag{83}$$

and

$$\varphi_\tau(x) = \int dy\, C_\tau(x-y)\, \frac{\delta\Gamma}{\delta\underline{\varphi}_\tau(y)}\,,\;\; c^a(x) = -\int dy\, S(x-y)\, \frac{\delta\Gamma}{\delta\underline{\bar{c}}^a(y)}\,,\;\; \bar{c}^a = \int dy\, \frac{\delta\Gamma}{\delta\underline{c}^a(y)}\, S(y-x)\,. \tag{84}$$

4.2 Flow Equations and Renormalizability of Vertex functions

In this section we shortly comment on flow equations for proper vertex functions. Such FE have been analysed previously in [KKSc] for ϕ_4^4-theory, to prove analyticity statements in Minkowski space. They have been derived and applied before in the literature, see e.g. [BAM1, Wet]. Writing (70, 76, 78) with general Λ instead of $\Lambda = 0$ we may derive FE similarly as in the previous chapter by deriving w.r.t. Λ. Deriving (76) we obtain

$$\partial_\Lambda \tilde{W}^{\Lambda,\Lambda_0}(K,\xi) \;=\; \partial_\Lambda P^{\Lambda,\Lambda_0}(K) \;-\; \partial_\Lambda \tilde{L}^{\Lambda,\Lambda_0}(\varphi_\tau, c, \bar{c})\,, \tag{85}$$

and (78) then implies

$$\partial_\Lambda \tilde{\Gamma}^{\Lambda,\Lambda_0} \;+\; \partial_\Lambda \tilde{W}^{\Lambda,\Lambda_0} = 0\;. \tag{86}$$

Combining both equations and using the FE derived from (48) for the functional $\tilde{L}^{\Lambda,\Lambda_0}$ we obtain the FE for $\tilde{\Gamma}^{\Lambda,\Lambda_0}$:

$$\partial_\Lambda \tilde{\Gamma}^{\Lambda,\Lambda_0}(\underline{\varphi}_\tau, \underline{\bar{c}}, \underline{c}) \;-\; \frac{1}{2}\sum_\tau \int_p \underline{\varphi}_\tau(p)\, \partial_\Lambda (C_\tau^{\Lambda,\Lambda_0}(p))^{-1} \underline{\varphi}_\tau(-p) \;+\; \int_p \underline{\bar{c}}^a(p)\, \partial_\Lambda (S^{\Lambda,\Lambda_0}(p))^{-1} \underline{c}^a(-p)$$

$$= \hbar\, (\partial_\Lambda \Delta(\Lambda,\Lambda_0))\, \tilde{L}^{\Lambda,\Lambda_0}(\varphi_\tau, c, \bar{c})\;. \tag{87}$$

The functional on the r.h.s. has to be viewed as depending on the (classical) fields $\underline{\varphi}_\tau,\ \underline{\bar{c}},\ \underline{c}$. In momentum space the fields $\varphi_\tau,\ \bar{c},\ c$ are given in terms of those through

$$\varphi_\tau(p) = (2\pi)^4\, C_\tau^{\Lambda,\Lambda_0}(p)\, \frac{\delta\tilde{\Gamma}^{\Lambda,\Lambda_0}}{\delta\underline{\varphi}_\tau(-p)}\,,\;\; c^a(p) \;= -(2\pi)^4\, S^{\Lambda,\Lambda_0}(p)\, \frac{\delta\tilde{\Gamma}^{\Lambda,\Lambda_0}}{\delta\underline{\bar{c}}^a(-p)}\,,$$

$$\bar{c}^a(p) \;=\; (2\pi)^4\, S^{\Lambda,\Lambda_0}(p)\, \frac{\delta\tilde{\Gamma}^{\Lambda,\Lambda_0}}{\delta\underline{c}^a(-p)}$$

corresponding to (84). The r.h.s of (87) is expressed in terms of $\underline{\varphi}_\tau,\ \underline{\bar{c}},\ \underline{c}$ [17] using the following relations (and the chain rule)

$$(2\pi)^{-4}\, (C_\tau^{\Lambda,\Lambda_0}(p))^{-1} \underline{\varphi}_\tau(p) \;=\; -\,\frac{\delta\tilde{L}^{\Lambda,\Lambda_0}}{\delta\varphi_\tau(-p)} \;+\; \frac{\delta\tilde{\Gamma}^{\Lambda,\Lambda_0}}{\delta\underline{\varphi}_\tau(-p)}\,,$$

[17] Note that $\Delta(\Lambda,\Lambda_0)$ in (87) is still the one in terms of the fields $\varphi_\tau,\ \bar{c},\ c$.

$$(2\pi)^{-4}\,(S^{\Lambda,\Lambda_0}(p))^{-1}\underline{c}^a(p) \;=\; \frac{\delta \tilde{L}^{\Lambda,\Lambda_0}}{\delta \bar{c}^a(-p)} \;-\; \frac{\delta \tilde{\Gamma}^{\Lambda,\Lambda_0}}{\delta \underline{\bar{c}}^a(-p)}\,, \tag{88}$$

$$(2\pi)^{-4}\,(S^{\Lambda,\Lambda_0}(p))^{-1}\underline{\bar{c}}^a(p) \;=\; -\,\frac{\delta \tilde{L}^{\Lambda,\Lambda_0}}{\delta c^a(-p)} \;+\; \frac{\delta \tilde{\Gamma}^{\Lambda,\Lambda_0}}{\delta \underline{c}^a(-p)}\,.$$

The inverted propagators appearing in (87, 88) remain only at the tree level, they cancel at loop order ≥ 1.

Considering first the functional without insertions we may again inductively bound the functions $\partial^w \Gamma^{\Lambda,\Lambda_0}_{l,n}$ proceeding as in Ch.3 upwards in l (note the factor of $\hbar$ on the r.h.s.), for given l upwards in $|n|$, and for given l, $|n|$ downwards in the number of momentum derivatives. The induction starts from the tree order vertex functional

$$\Gamma^{\Lambda,\Lambda_0}_{l=0} \;=\; \frac{1}{2}\sum_\tau \int_p \underline{\varphi}_\tau(p)\,(C^{\Lambda,\Lambda_0}_\tau(p))^{-1}\underline{\varphi}_\tau(-p) \;-\; \int_p \underline{\bar{c}}^a(p)\,(S^{\Lambda,\Lambda_0}(p))^{-1}\underline{c}^a(-p)$$

$$+\,(\Gamma^{\Lambda,\Lambda_0}_3 \;+\; \Gamma^{\Lambda,\Lambda_0}_4)_{l=0} \;+\; L^0_{irr}|_{l=0}\;. \tag{89}$$

The tree level three and four point functions from the third term are given in App.A, the last term is the tree level contribution to the irrelevant extension of L^0 in (107, 108). Imposing b.c. analogous to those imposed on the CAG from Ch.3.2 in A1), A2) we may then derive the bounds

Proposition 5 :

$$|\partial^w \Gamma^{\Lambda,\Lambda_0}_{l,n}(\vec{p})| \leq\; (\Lambda+m)^{4-|n|-|w|}\,\mathcal{P}_1(log\frac{\Lambda+m}{m})\,\mathcal{P}_2(\frac{|\vec{p}|}{\Lambda+m}) \tag{90}$$

with the same comments as for Proposition 1.

We again skip the proof. Finally we note that to obtain the analogous renormalizability statements for vertex functions with one insertion the FE (87) has to be derived w.r.t. the corresponding source. Again a FE linear in terms of the inserted vertex functions, but involving also the noninserted ones, emerges. Its solutions are bounded in the same way as the corresponding CAG from Ch.3.

Since the analysis of the STI is more transparent in terms of the vertex functions, the renormalization conditions will be imposed on those. We may then directly infer the finiteness of the theory from the results of this section. We could also calculate from the b.c. on the vertex functions those for the CAG, which then also satisfy A1),A2) and conclude on the finiteness by Ch.3, so that we might have skipped this section altogether, paying instead more attention on how to calculate b.c. on L from those for Γ and vice versa. Generally speaking it seems to us that FE for vertex functions are useful in their own right. Nevertheless the CAG should perhaps be viewed as the "primary objects" of interest, since the

FE for them takes a closed functional form. This closed form is of fundamental importance for the analysis of the linear relations among the STI and thus crucial for the proof of the Theorem and in particular Lemma 2 below.

4.3 Violated Slavnov-Taylor Identities for the bare functional L^0

In this section we use again the abbreviations

$$\Delta \,=\, \Delta(0,\Lambda_0)\,,\ L \,=\, L^{0,\Lambda_0},\ \tilde{L} \,=\, \tilde{L}^{0,\Lambda_0},\ L^0 \,=\, L^{\Lambda_0,\Lambda_0},\ \tilde{L^0} \,=\, \tilde{L}^{\Lambda_0,\Lambda_0},\ L_1^0 \,=\, L_1^{\Lambda_0,\Lambda_0}\ . \tag{91}$$

Our starting point are the VSTI (75). By commuting the functional differential operator appearing on the rhs of (75) with the renormalization group flow we will obtain the VSTI in terms of L^0. We introduce some further abbreviations:

$$\frac{\delta}{\delta R^a(x)} \,=\, \frac{1}{\alpha}\partial_\nu \frac{\delta}{\delta A_\nu^a(x)} \,-\, m\frac{\delta}{\delta B^a(x)}\,,\quad X \,=\, \langle Dc^a,\, (\frac{1}{\alpha}\partial_\nu A_\nu^a - m\, B^a)\rangle\,, \tag{92}$$

$$Y \,=\, \langle c^a,\, \sigma_{0,\Lambda_0}\frac{\delta}{\delta R^a}\rangle \,-\, \sum_\tau \langle \varphi_\tau,\, D_\tau \frac{\delta}{\delta\gamma_\tau}\rangle \,+\, \langle \bar{c}^a,\, D\,\frac{\delta}{\delta\omega^a}\rangle\,.$$

Now we can write (75) in the form

$$L_1 \,=\, e^{\frac{1}{\hbar}\tilde{L}}\,(X+\hbar Y)\, e^{-\frac{1}{\hbar}\tilde{L}}|_{\xi\equiv 0}\ . \tag{93}$$

The last two factors may be rewritten as (remember (48))

$$(X+\hbar Y)\, e^{-\frac{1}{\hbar}\tilde{L}} \,=\, e^{\frac{1}{\hbar}I}\, e^{\hbar\,\Delta}\, e^{-\hbar\,\Delta}\,(X+\hbar Y)\, e^{\hbar\,\Delta}\, e^{-\frac{1}{\hbar}\tilde{L^0}} \tag{94}$$

$$=\, e^{\frac{1}{\hbar}I}\, e^{\hbar\,\Delta}\Big(X+\hbar Y - \hbar[\Delta, X+\hbar Y] + \frac{\hbar^2}{2}[\Delta,[\Delta, X+\hbar Y]]\Big)\, e^{-\frac{1}{\hbar}\tilde{L^0}}\ .$$

We have to calculate the commutators

$$[\Delta, Y] \,=\, -\langle\frac{\delta}{\delta\bar{c}^a},\, \sigma_{0,\Lambda_0}S\,\frac{\delta}{\delta R^a}\rangle \,-\, \sum_\tau\langle\frac{\delta}{\delta\varphi_\tau},\, \sigma_{0,\Lambda_0}\frac{\delta}{\delta\gamma_\tau}\rangle \,+\, \langle\frac{\delta}{\delta c^a},\, \sigma_{0,\Lambda_0}\,\frac{\delta}{\delta\omega^a}\rangle\,, \tag{95a}$$

$$[\Delta, X] \,=\, \langle c^a,\, \sigma_{0,\Lambda_0}\frac{\delta}{\delta R^a}\rangle \,-\, \langle\frac{\delta}{\delta\bar{c}^a},\, \sigma_{0,\Lambda_0}(\frac{1}{\alpha}\partial_\nu A_\nu^a \,-\, m\, B^a)\rangle\,, \tag{95b}$$

$$[\Delta,[\Delta, X]] \,=\, -2\,\langle\frac{\delta}{\delta\bar{c}^a},\, \sigma_{0,\Lambda_0}S\,\frac{\delta}{\delta R^a}\rangle\ . \tag{95c}$$

From these relations we obtain

$$(X+\hbar Y)\, e^{-\frac{1}{\hbar}\tilde{L}} \,=\, e^{\frac{1}{\hbar}I}\, e^{\hbar\,\Delta}\Big\{\langle c^a,\, D\,(\frac{1}{\alpha}\partial_\nu A_\nu^a \,-\, m\, B^a)\rangle \,-\, \langle\frac{\delta\tilde{L^0}}{\delta\bar{c}^a},\, \sigma_{0,\Lambda_0}(\frac{1}{\alpha}\partial_\nu A_\nu^a \,-\, m\, B^a)\rangle \tag{96}$$

$$+\sum_\tau\langle\varphi_\tau,\,D_\tau\frac{\delta\tilde L^0}{\delta\gamma_\tau}\rangle\;-\;\langle\bar c^a,\,D\,\frac{\delta\tilde L^0}{\delta\omega^a}\rangle\;+\;\sum_\tau\langle\frac{\delta\tilde L^0}{\delta\varphi_\tau},\,\sigma_{0,\Lambda_0}\frac{\delta\tilde L^0}{\delta\gamma_\tau}\rangle\;-\;\langle\frac{\delta\tilde L^0}{\delta c^a},\,\sigma_{0,\Lambda_0}\frac{\delta\tilde L^0}{\delta\omega^a}\rangle\Big\}e^{-\frac{1}{\hbar}\tilde L^0}\;.$$

Note that due to the form of $\tilde L^0$ the contribution

$$\hbar\sum_\tau\langle\frac{\delta}{\delta\varphi_\tau},\,\sigma_{0,\Lambda_0}\frac{\delta}{\delta\gamma_\tau}\rangle\,\tilde L^0\;-\;\hbar\,\langle\frac{\delta}{\delta c^a},\,\sigma_{0,\Lambda_0}\frac{\delta}{\delta\omega^a}\rangle\,\tilde L^0$$

vanishes and thus may be omitted in the parentheses in (96). On the other hand using (93, 74) we can also express $(X+\hbar Y)\,e^{-\frac{1}{\hbar}\tilde L}|_{\xi\equiv 0}$ in terms of L_1^0:

$$(X+\hbar Y)\,e^{-\frac{1}{\hbar}\tilde L}|_{\xi\equiv 0}\;=\;L_1\,e^{-\frac{1}{\hbar}\tilde L}|_{\xi\equiv 0} \tag{97}$$

$$=\;-\hbar\,\frac{d}{d\chi}e^{-\frac{1}{\hbar}L_\chi}|_{\chi=0}=(-\hbar\frac{d}{d\chi}\,e^{\frac{1}{\hbar}I}\,e^{\hbar\Delta}\,e^{-\frac{1}{\hbar}L_\chi^0}|_{\chi=0})=e^{\frac{1}{\hbar}I}\,e^{\hbar\Delta}\,L_1^0\,e^{-\frac{1}{\hbar}L^0}\;.$$

Remember that $\tilde L|_{\xi\equiv 0}\;=\;L_\chi|_{\chi=0}\;=\;L$. Equality of (96) for $\xi\equiv 0$ and (97) and invertibility of $\exp\hbar\Delta$ (in perturbation theory) now obviously give

$$\langle c^a,\,D\,(\frac{1}{\alpha}\partial_\nu A_\nu^a\;-\;m\,B^a)\rangle\;-\;\langle\frac{\delta L^0}{\delta\bar c^a},\,\sigma_{0,\Lambda_0}(\frac{1}{\alpha}\partial_\nu A_\nu^a\;-\;m\,B^a)\rangle \tag{98}$$

$$+\sum_\tau\langle\varphi_\tau,\,D_\tau L^0_{\gamma_\tau}\rangle\;-\;\langle\bar c^a,\,D\,L^0_{\omega^a}\rangle\;+\;\sum_\tau\langle\frac{\delta L^0}{\delta\varphi_\tau},\,\sigma_{0,\Lambda_0}L^0_{\gamma_\tau}\rangle\;-\;\langle\frac{\delta L^0}{\delta c^a},\,\sigma_{0,\Lambda_0}L^0_{\omega^a}\rangle\;=\;L_1^0\;.$$

(98) is the VSTI for the bare functional L^0. It turns out that it plays -unexpectedly- a prominent role in the analysis of how the STI can be restored. Since we impose renormalization conditions in momentum space we also express (98) through the Fourier transformed fields (using the conventions from Ch.2)

$$L_1^0\;=\;\int_p c^a(p)(p^2+\alpha\,m^2)\{-\frac{i}{\alpha}p_\nu A_\nu^a(-p)\;-\;m\,B^a(-p)\} \tag{99}$$

$$-\,(2\pi)^4\int_p\frac{\delta L^0}{\delta\bar c^a(p)}\,\{\frac{i}{\alpha}p_\nu A_\nu^a(p)\;-\;m\,B^a(p)\}\,\sigma_{0,\Lambda_0}(p^2)$$

$$+\int_p A_\mu^a(p)[(p^2+m^2)\delta_{\mu\nu}+\frac{1-\alpha}{\alpha}p_\mu p_\nu]L^0_{\gamma_\nu^a(p)}\;+\;\int_p h(p)(p^2+M^2)L^0_{\gamma(p)}$$

$$+\int_p B^a(p)(p^2+\alpha m^2)L^0_{\gamma^a(p)}-\int_p\bar c^a(p)(p^2+\alpha m^2)L^0_{\omega^a(p)}$$

$$+\,(2\pi)^4\int_p\sigma_{0,\Lambda_0}(p^2)\Big\{\frac{\delta L^0}{\delta A_\lambda^a(p)}L^0_{\gamma_\lambda^a(-p)}\;+\;\frac{\delta L^0}{\delta h(p)}L^0_{\gamma(-p)}\;+\;\frac{\delta L^0}{\delta B^a(p)}L^0_{\gamma^a(-p)}\;-\;\frac{\delta L^0}{\delta c^a(p)}L^0_{\omega^a(-p)}\Big\}\;.$$

4.4 Choice of Renormalization Conditions and Restoration of the Slavnov-Taylor-Identities

We have derived the STI in the previous two subsections for all three functionals $\Gamma,\ L,\ L^0$. In fact the L-functional is only needed as a connecting link between the other two. As we mentioned before this threefold description will be required to recognize the linear interdependences among the STI projected onto the relevant parts of the various functionals. For this purpose we also need termwise equivalence relations among the relevant parts of Γ and L^0. These termwise equivalence relations are simplified, if *we assume that the renormalization conditions for the functionals* Γ *or* L *are chosen such that*:

$$\kappa := \frac{\delta\Gamma}{\delta\underline{h}(x)}|_{\underline{\Phi}\equiv 0} \,=\, 0 \Longleftrightarrow \frac{\delta L}{\delta h(x)}|_{\Phi\equiv 0} \,=\, 0\,. \tag{100}$$

The condition (100) on the absence of tadpoles, although probably not indispensable, simplifies the subsequent formulae, and it is not really a physical restriction, but rather one on the parametrization of the theory. Here and in the following we use the shorthand notation

$$\partial^w\delta^n_\Phi F|_0$$

to denote the derivative of the functional F (which might be L or Γ) w.r.t. n fields Φ, evaluated at $\Phi\equiv 0$, followed by removing the global δ-function and performing the derivatives ∂^w. When we write

$$\partial^w\delta^n_\Phi F|_{0,0}$$

we set in addition all momenta to 0 afterwards, and

$$\partial^w\delta^n_\Phi F|_{0,0,l} \tag{101}$$

is the l-th order coefficient in the loop expansion of the previous expression. We now state

Lemma 1: Under the assumption (100) we have:
If for given l, n, w and for all l', n', w' with $l' < l$ and $(n',\, w') \subseteq (n,\, w)$ or with $l' = l$ and $(n',\, w') \subset (n,\, w)$ we have $\partial^{w'}\delta^{n'}_{\underline{\Phi}}\Gamma_1|_{0,0,l'} \,=\, 0$, then[18]

$$\partial^w\delta^n_{\underline{\Phi}}\Gamma_1|_{0,0,l} \,=\, 0 \Longleftrightarrow \partial^w\delta^n_\Phi L_1|_{0,0,l} \,=\, 0\,. \tag{102}$$

Proof: (102) follows from (83, 84) on noting that all propagators are finite and nonvanishing and that all possible factorizations appearing when we apply the chain rule in going from

[18] We use the set theoretic relations for the multiindices n, w though strictly speaking they are sequences. The symbol $\subset$ means by definition strict inclusion.

the L- to the Γ-functions or vice versa vanish due to the conditions on the lower dimension terms. $\blacksquare$

Lemma 1 suggests that we satisfy the STI for the relevant terms proceeding upwards in the number of fields and momentum derivatives. The subsequent comparison of the VSTI for L (75) and L^0 (98) shows that we also should proceed upwards in the number of loops. Before proceeding to the termwise comparison it is instructive to note quite generally that from

$$L_1 \;=\; \frac{d}{d\chi} L_\chi|_{\chi=0} \quad \text{and} \quad L_1^0 \;=\; \frac{d}{d\chi} L_\chi^0|_{\chi=0} \tag{103}$$

it follows similarly as in (97) that

$$L_1\, e^{-\frac{1}{\hbar}(L+I)} \;=\; e^{\hbar\Delta} L_1^0\, e^{-\frac{1}{\hbar}L^0}\,, \tag{104}$$

and from the (perturbative) invertibility of $e^{\hbar\Delta}$ we then obtain the relation

$$L_1 \;=\; 0 \Longleftrightarrow L_1^0 \;=\; 0\ . \tag{105}$$

Our goal is to arrange for renormalization conditions such that the relevant terms in Γ_1 vanish, proceeding inductively in the number of loops l. These relevant terms are listed in App.C, $(I - XXIX)$. By the statements from Ch.2 and 4.2 and App.C there are no nonvanishing relevant terms in Γ_1 and L_1^0 at the tree level in the limit $\Lambda_0 \to \infty$ (this limit remains formal before we have stated how to renormalize the theory in agreement with the STI). Since in the relevant part of the VSTI there are contributions stemming from $\dot{\sigma}$ (106) for finite Λ_0 which might conspire to give finite contributions in the VSTI when combining with divergent terms, our strategy is to compensate for them by introducing irrelevant terms in the bare action L^0. In this respect it is important to note that the termwise identities (I-XXIX) take the same form for Γ and L^0 apart from the crucial fact that

i) L^0 contains only those irrelevant terms we are going to introduce explicitly,

and from the fact that

ii) there appear additional terms in (98) as compared to (82) which just replace those 0-loop terms, excluded in L^0 by its definition[19], so that as a consequence the termwise identities look as before (when ignoring the irrelevant terms).

We will shortly denote the relevant terms in L^0 by adding a sub- or superscript 0 to the corresponding term appearing in Γ. In the same way we denote $(I - XXIX)$ written for L^0 as $(I^0 - XXIX^0)$. In a number of STI the irrelevant terms introduced in L^0 below (107, 108) will make appearance, namely in III, V, VII, $VIII$. For those terms the STI for L^0

[19]these terms contribute only when performing up to three field derivatives.

are rewritten explicitly in App.C including these terms. We use similar notation as in App.A and App.C, in particular the shorthand

$$\dot\sigma \;:=\; \dot\sigma_{0,\Lambda_0}(0) \;:=\; \frac{d\sigma_{0,\Lambda_0}(p^2)}{dp^2}|_{p^2=0} \;=\; -\frac{\alpha m^4 + (1+\alpha) m^2 M^2}{\Lambda_0^6} \tag{106}$$

and add the following contribution[20] to L^0

$$\begin{aligned} L^0_{irr} \;=\; \int_p \int_q \Big\{ & \epsilon^{rst} A^r_\mu(p) A^s_\nu(q) B^t(-p-q)\,[\,\delta_{\mu\nu}\,(p^2-q^2)\, i^{AAB}_{10} \,+\, (p_\mu p_\nu - q_\mu q_\nu) i^{AAB}_{20}\,] \\ & + \bar c^r(p) c^r(q) h(-p-q)\,[\,p^2\, i^{\bar c c h}_{10} \,+\, q^2\, i^{\bar c c h}_{20} \,+\, pq\, i^{\bar c c h}_{30}\,] \\ & + \epsilon^{rst} \bar c^r(p) c^s(q) B^t(-p-q)\,[\,p^2\, i^{\bar c c B}_{10} \,+\, q^2\, i^{\bar c c B}_{20} \,+\, pq\, i^{\bar c c B}_{30}\,]\;. \end{aligned} \tag{107}$$

We have presented L^0 directly in momentum space, where we perform the analysis of the STI. The letter i was chosen to remind of 'irrelevant', and we listed all terms of the respective field content allowed by the global symmetries, which are of second order in the momenta. The constants $i^{\cdots}$ will be chosen as follows:

$$\begin{gathered} 2\, i^{AAB}_{10} m\, R^0_4 \;=\; -\dot\sigma\, \delta m_0^2\, g\, R^0_2\,, \quad i^{AAB}_{20} \;=\; 0\,, \\ m(i^{\bar c c B}_{10} \,-\, i^{\bar c c B}_{30}) \;=\; -\dot\sigma\,[m\, F^{\bar c c B}_0 \,-\, \Sigma^{BB}_0 \frac{1}{2} g\, R^0_6]\,, \quad \frac{m}{2}\, i^{\bar c c h}_{30} \;=\; \dot\sigma\, \Sigma^{hh}_0 \,\frac{g}{2}\, R^0_3\,, \\ \frac{m}{2}(2 i^{\bar c c h}_{10} \,-\, i^{\bar c c h}_{30}) \;=\; -\dot\sigma [m\, F^{\bar c c h}_0 \,+\, \Sigma^{BB}_0\, \frac{g}{2}\, R^0_5]\,, \quad \frac{m}{2}(2 i^{\bar c c h}_{20} \,-\, i^{\bar c c h}_{30}) \;=\; -2\,\dot\sigma F^{BBh}_0\, m\, R^0_4\,, \\ m\, R^0_4 (2\, i^{\bar c c B}_{10} \,-\, i^{\bar c c B}_{30}) = \dot\sigma\, \Sigma^{\bar c c}_0\, g\, R^0_7\,, \quad i^{\bar c c B}_{20} \;=\; -\dot\sigma F^{\bar c c B}_0\,. \end{gathered} \tag{108}$$

These relations are written in terms of the linear combinations which appear in the respective STI and are needed to verify them. By the general results of Ch.3 and Ch.4.2 the theory stays finite when adding such "irrelevant" dimension 5 terms to the bare action, under the condition that these terms can be bounded by $\Lambda_0^{-1}\mathcal{P}_1(log(\Lambda_0/m))\,\mathcal{P}_2(\frac{|\vec p|}{\Lambda_0})$. If the relevant terms appearing in (108) obey A1)(38), this bound is obvious from the fact that $\dot\sigma = O(\Lambda_0{}^{-6})$. After this modification of the bare action we may state our

Induction hypothesis: For $l \ge 1$ and all $l' \le l-1$

i) we assume that the theory to order l' has been renormalized according to A1)(38) and A2) for the Γ (or equivalently for the L) functional.

ii) Furthermore we assume

$$\partial^w \delta^n_{\underline\Phi} \Gamma_1|_{0,0,l'} \;=\; 0\,,\;\; \partial^w \delta^n_{\Phi} L^0_1|_{0,0,l'} \;=\; 0\,,\;\; \text{for}\;\; (n,w)\;\; \text{with}\;\; |n|+|w| \le 5\;. \tag{109}$$

[20]We remark that when working with a regulator as in footnote 7, we could spare the detour (107,108), because then $\dot\sigma$ would be zero.

iii) Finally we assume that

$$|\partial^w \delta_\Phi^n L_1^{\Lambda_0,\Lambda_0}|_{0,l'}| \le O(\Lambda_0^{5-|n|-|w|})\, \mathcal{P}_1(log\frac{\Lambda_0}{m})\, \mathcal{P}_2(\frac{|\vec{p}|}{\Lambda_0})\ , \quad \text{for } (n,w) \text{ with } |n|+|w|>5\ . \tag{110}$$

All these statements are fulfilled at the tree level by our assumptions on the tree level action. The rest of this section is devoted to prove the

Theorem: *The induction hypothesis holds at loop order l.*

Proof: At loop order l we first prove the crucial

Lemma 2: For given (n,w) with $|n|+|w|\le 5$ under the assumptions (100, 109) and if

$$\partial^{w'}\delta_\Phi^{n'} L_1|_{0,0,l} = 0\ , \quad \partial^{w'}\delta_\Phi^{n'} L_1^0|_{0,0,l} = 0 \ \text{ for } (n',w')\subset(n,w) \text{ and } n'\subset n \tag{111}$$

the following equality holds:

$$\partial^w \delta_\Phi^n L_1|_{0,0,l} = \partial^w \delta_\Phi^n L_1^0|_{0,0,l}\ . \tag{112}$$

Proof: Due to the induction assumption ii), Lemma 1 and (111) we find

$$\Big[(-\hbar)\frac{d}{d\chi}\partial^w \delta_\Phi^n\, e^{-\frac{1}{\hbar}L_\chi}\Big]|_{0,0,\chi=0,l} = \partial^w \delta_\Phi^n\, L_1|_{0,0,l} \tag{113}$$

noting that factorized terms give vanishing contribution, since $|n|+|w|\le 5$. On the other hand we also obtain (cf. (104))

$$\Big[(-\hbar)\frac{d}{d\chi}\partial^w \delta_\Phi^n\, e^{-\frac{1}{\hbar}L_\chi}\Big]|_{\Phi\equiv0,\chi=0,\,l} = \Big[\partial^w \delta_\Phi^n\,(e^{\hbar\Delta}\, L_1^0\, e^{-\hbar\Delta}\, e^{-\frac{1}{\hbar}L})\Big]|_{\Phi\equiv0,\,l}\ . \tag{114}$$

Note that here we do not yet restrict to vanishing momenta $\vec{p}$, but assume that the momenta of the fields appearing in the derivatives to be called $p_1,\dots p_{|n|}$ have been chosen nonexceptional [21]. Later we take $\vec{p}\to 0$ [22]. We may rewrite the term $e^{\hbar\Delta}\, L_1^0\, e^{-\hbar\Delta}$ as

$$e^{\hbar\Delta}\, L_1^0\, e^{-\hbar\Delta} = L_1^0 + \sum_{\nu=1}^{5}\frac{\hbar^\nu}{\nu\,!}[\Delta\,,\,L_1^0]^\nu\ , \tag{115}$$

with the definition

$$[\Delta\,,\,\cdot\,]^\nu := \underbrace{[\Delta\,,\,[\dots[\Delta\,,\,\cdot\,]\dots]]}_{\nu\ times}\ . \tag{116}$$

---

[21] i.e. no subsum vanishes

[22] We point out that (113) should strictly speaking also be viewed as being obtained first for nonexceptional $\vec{p}$, where correction terms appear, which then smoothly tend to 0 for $\vec{p}\to 0$, so that we need not pay attention to them.

In (115) we used that L_1^0 is of degree 5 in the fields. We may then define

$$P_1^0\, e^{-\frac{1}{\hbar}L} \;=\; (\, L_1^0 \,+\, \sum_{\nu=1}^{5} \frac{\hbar^\nu}{\nu\,!}\,[\Delta\,,\, L_1^0]^\nu\,)\, e^{-\frac{1}{\hbar}L}\,, \tag{117}$$

and recognize P_1^0 as given by the sum over the contributions from the connected amputated diagrams containing
i) exactly one vertex from L_1^0
ii) up to 5 vertices from $-L$, which are all directly linked to the vertex from L_1^0 via a propagator from Δ
iii) multiplied by the monomial in the fields produced by the derivatives from Δ acting on the respective term in $(-L)$, multiplied by the respective power of $\hbar$ and a combinatoric factor to be read from (117).
We now have to regard

$$\partial^w \delta_\Phi^n (P_1^0\, e^{-\frac{1}{\hbar}L})|_{\Phi\equiv 0,\, l}\ . \tag{118}$$

After performing the field and momentum derivatives and after splitting off the global $\delta(p_1 + \ldots + p_{|n|})$-function we let all momenta go to 0 so that then

$$\partial^w \delta_\Phi^n (P_1^0\, e^{-\frac{1}{\hbar}L})|_{0,0,\, l}\ . \tag{119}$$

is given by
the sum over all l-loop connected amputated diagrams containing exactly one vertex from L_1^0, $|n|$ external lines of the kind specified in δ_Φ^n, up to 5 vertices from $-L$ directly linked to the one from L_1^0 via a propagator, and weighed with a combinatoric factor as above. The functions are derived w.r.t. external momenta as indicated through ∂^w and taken at 0 external momenta in the end.

Note that the restriction on the momenta avoids the production of disconnected terms by momentum conservation. Now remembering (100) and the fact that L does not contain 0-loop two point functions we can use the induction hypothesis (109) and (111) to conclude that all contributions to (119) vanish apart from the term

$$\partial^w \delta_\Phi^n L_1^0|_{0,\,0,\,l} \;=\; \partial^w \delta_\Phi^n L_1^{\Lambda_0,\Lambda_0}|_{0,\,0,\,l}\,. \tag{120}$$

Any other contribution would require nonvanishing $\partial^{w'} \delta_\Phi^{n'} L_1^0|_{0,\,0,\,l'}$ with $l' < l$ or $(n', w') \subset (n, w)$ and $n' \subset n$. The term (120) then equals $\partial^w \delta_\Phi^n L_1|_{\Phi\equiv 0,\,0,\,l}$ by (113, 114) and subsequent comments. ∎

After these preparations we present the renormalization conditions at l-loop order, lower orders being already fixed by induction. This means we fix the 37 relevant terms of the theory

and the 7 normalization parameters R_i appearing in the BRS transformation at order $\hbar^{l}$ [23]:
A) We fix $\kappa = 0$ (100), and we choose freely in Γ the 8 terms[24]

$$\underline{\Sigma}_{trans}\,,\ \underline{\Sigma}_{long}\,,\ \dot{\underline{\Sigma}}^{BB},\ \dot{\underline{\Sigma}}^{\bar{c}c},\ \Sigma^{AB},\ F^{BBh},\ R_2,\ R_3\,. \tag{121}$$

This then fixes uniquely the corresponding terms in L. In fact we could interchange R_1 with $\underline{\Sigma}_{long}$, R_4 with Σ^{AB}, and/or F^{AAA} with R_2 in (121). We made the previous choice since it simplifies the check of the STI.
This means that we may choose freely all field normalizations with the exception of h[25], one global normalization for the BRS-transformations and the two couplings through F^{AAA} and F^{BBh}. Our simplifying assumption $\kappa = 0$ (100) is related to the freedom in choosing the vacuum expectation value of the Higgs field.
B) We fix in $\tilde{L}^0$ the following relevant terms:

$$R_6^0 \;=\; R_7^0 \;=\; R_2^0\,, \quad R_5^0 \;=\; \frac{(R_2^0)^2}{R_3^0}. \tag{122}$$

This means that R_6^0, R_7^0 are fixed to equal R_2^0, which in turn is uniquely given at l loop order by our free choice of R_2, and by lower loop order constants fixed before. Similarly R_5^0 is fixed through R_3^0 and R_2^0 at l loop order. Remember again that, by the FE for 1PI functions, an l-loop contribution depends only on lower loop order terms and the l-loop boundary condition for the term in question.
C) All those r_0-terms in L^0 having no tree correspondence are chosen equal to zero (11 terms to be read from App.A), i.e.

$$r_{20}^{hBA},\,\ldots,\,r_0^{\bar{c}c\bar{c}c} \;=\; 0\,. \tag{123}$$

D) Furthermore we fix in L_0 the following relevant terms

$$F_0^{BBA} \;=\; -\frac{R_3^0}{2R_2^0}\,F_{10}^{hBA}\,,\ \ F_0^{\bar{c}cB} \;=\; -\frac{R_3^0}{R_2^0}\,F_0^{\bar{c}ch}\,,\ \ F_0^{AAhh} \;=\; \frac{R_5^0}{R_3^0}\,F_{10}^{AABB}\,. \tag{124}$$

More precisely (124) should be understood as follows: The F-terms on the rhs in (124) will be uniquely fixed as functions of (a subset of) the ones fixed previously in A)-C). Then we fix each of the three terms on the lhs as a function of those on the rhs.

Finally also the remaining 18 relevant constants will be uniquely fixed as functions of the previous ones in our sweep through the STI. Since 17 relevant terms (those from B)-D)) are fixed on the wrong side, namely in L^0, one may wonder, how we will get a finite theory in the

[23] We mostly leave out the index l of the loop order for readibility in the rest of this subsection.
[24] cf. App.A (137) for the notation
[25] remember that h, B^a stem from the same complex scalar doublet (5)

end. The tool to achieve this will in fact again be the STI, once we know they are satisfied. This is not unexpected from the traditional use made of the STI in renormalization proofs. Now we first satisfy a subset of the STI (I-VIII) containing only up to three field derivatives by choosing appropriately

$$\delta m^2[\Sigma^{\bar{c}c}, R_1]\,(I_a),\ \Sigma^{BB}[\Sigma^{\bar{c}c}, R_4]\,(II_a)\,,\ R_1\,(I_b)\,,\ R_4\,(II_b)\,,\ F^{AAA}[\dot{\sigma}\,\delta m^2]\,(III_b)\,, \tag{125}$$
$$F_1^{hBA}[R_5,\,\dot{\sigma}(F^{\bar{c}ch},\,\Sigma^{BB})]\,(VII_c)\,,\ F^{AAh}[F_1^{hBA}](VI_b)\,,\ F_1^{\bar{c}cA}[F^{BBA}, r_2^{\bar{c}cA}]\,(IV_b)\,,$$
$$F^{\bar{c}ch}[F^{AAh}, F_1^{hBA}, r_2^{hBA}]\,(VI_a)\,,\ \Sigma^{\bar{c}c}[R_7, F^{\bar{c}cB}]\,(VIII_a)\,,\ \Sigma^{hh}[R_5, \Sigma^{BB}, F^{\bar{c}ch}]\,(VII_a)\,,$$
$$\dot{\Sigma}^{hh}[F_1^{hBA}, \dot{\sigma}\,\Sigma^{hh}]\,(VII_b)\ .$$

We wrote in brackets the STI which is satisfied by the respective choice of a renormalization constant and in square brackets the other relevant constants at *loop order* l, on which this choice depends. In the square brackets we omitted the terms from (121), which are freely chosen, and R_1 and R_4, which by (125) depend on such terms only. Note however that I_b and II_b cannot be solved for R_1 and R_4 depending only on such terms, before we know that I_a and II_a hold. Therefore we indicated the dependence on R_1 and R_4 in the first two terms. At this stage R_1 and R_4 can already be seen to be finite. All other terms, depending on constants fixed on the wrong side, might diverge with Λ_0. We come back to the finiteness problem later and first convince ourselves that the system (125) is consistent, i.e. solvable. This is a problem only, if a term is present before and within square brackets at the same time, when we successively replace each term within square brackets by those on which it depends at l loop.[26] Checking the list we find that this happens only for $F^{\bar{c}ch}$, which, when substituting F_1^{hBA} from (125), depends on itself. Solving for $F^{\bar{c}ch}$ it appears with a coefficient $1/\alpha\,+\,\dot{\sigma}\,m^2(R_4/R_1)$. Since we know that R_1, R_4 are finite, this coefficient does not vanish for Λ_0 large.[27] As a result we may replace $F^{\bar{c}ch}[F_1^{hBA}, r_2^{hBA}]$ by

$$F^{\bar{c}ch}[r_2^{hBA}]\,. \tag{126}$$

After this change one rapidly realizes the solvability of (125).

Now we impose renormalization conditions for the remaining 6 relevant terms by satisfying the following relations among $(I-XXIX)$ for Γ.

$$F^{BBBB}\,(X)\,,\ F^{BBhh}\,(XX)\,,\ F^{hhhh}\,(XIX)\,,\ F^{hhh}\,(IX)\,,\ F_1^{AABB}\,(XIII_2)\,,\ F_1^{AAAA}\,(XIV_c)\ . \tag{127}$$

[26] E.g. at l loop R_6 depends on R_2 only by (122), whereas F^{BBA} depends on R_3, R_2, F_1^{hBA} by (124).

[27] α is supposed to be finite, but $\alpha \to \infty$ may be taken after $\Lambda_0 \to \infty$.

The order is important for the first four terms, for the last two it is arbitrary. Again we wrote in parentheses the relation which is satisfied by and which fixes the respective renormalization condition.

At this stage the 37 + 7 relevant parameters are completely fixed. All the remaining relations among the STI will now be verified for L^0. Since there are no dimension 3 terms left, we start with the dimension 4 terms which have not yet been verified. IV_a^0 is the only relation left among $(I^0 - VIII^0)$: Using (123) it takes the form

$$2m\, R_4^0\, F_0^{BBA} + g/2\, R_6^0\, \Sigma_0^{AB} + 1/\alpha\, F_0^{\bar{c}cB} = 0\,. \tag{128}$$

From (125) we know VI_a and VI_b to be true. Lemma 2 then implies VI_a^0 and VI_b^0 to be true as well. These relations together with (122, 123, 124) then allow to verify (128). Now

$$XI^0\,,\ XII^0\,,\ XIII_1^0 \tag{129}$$

are the last relations of dimension ≤ 4 to be analysed. They follow directly from (122, 123, 124). By Lemma 2 we pass from L_1^0 to L_1 for XI, XII, $XIII_1$.

Therefore Lemma 1 now tells us that all terms in Γ_1 of dimension ≤ 5 vanish iff they vanish in L_1, and Lemma 2 tells us that all terms in L_1^0 of dimension ≤ 5 vanish iff they vanish in L_1.

Among the relations containing 4 or more field derivatives the following ones

$$\begin{gathered} XIV_a^0\,,\ XIV_b^0\,,\ XIV_d^0\,,\ XIV_e^0\,,\ XV_{1a}^0\,,\ XV_{1b}^0\,,\ XV_{2a}^0\,,\ XV_{2b}^0\,,\ XV_{2c}^0\,,\ XVI_a^0\,,\ XVI_b^0\,, \\ XVI_c^0\,,\ XVII_a^0\,,\ XVII_b^0\,,\ XVIII_a^0\,,\ XVIII_b^0\,,\ XVIII_c^0\,,\ XXI^0\,,\ XXII^0 - XXIX^0 \end{gathered} \tag{130}$$

remain to be verified. Only those written in (131) are not immediately obvious from (122, 123, 124). They can be verified using the relations we wrote in parentheses

$$XV_{1a}^0\,(122,\, VI_b^0,\, XIII_2^0)\,,\ XVII_a^0\,(124,\, XIII_2^0,\, VI_b^0)\,. \tag{131}$$

We have not yet checked the following five relations of dimension 5 involving three fields only : III_a, V, VII_d, $VIII_b$, $VIII_c$, which are the most delicate ones. They contain terms multiplied by $\dot{\sigma}$. For III_a this is true when inserting F^{AAA} from III_b. We first forget about these corrections and at the same time about the modification of L_0 by (107, 108) and convince ourselves that the 5 STI are fulfilled in this case. To do so we use the following relations

$$I_b^0,\ II_b^0,\ IV_b^0,\ VII_c^0\,. \tag{132}$$

First we can verify V^0 using (124, 122) and VII_c^0. Next we regard III_a^0 and realize that it amounts to show that

$$g\, R_2^0(1+\underline{\Sigma}_{\mathrm{long},0}) \;=\; F_{1,0}^{\bar{c}cA}\,. \tag{133}$$

The lhs equals $g\,(R_2^0/R_1^0)(1+\dot{\underline{\Sigma}}^{\bar{c}c})$ by I_b^0, the rhs equals $-g/m\, R_2^0\, \Sigma_0^{AB} - 4\, R_4^0\, F_0^{BBA}$ by IV_b^0. Using II_b^0 for the first and V^0 for the second term we recognize now that (133) holds. VII_d^0 follows directly from (122), and $VIII_b^0$ follows similarly as III_a^0 from IV_b^0, II_b^0 and V^0, $VIII_c^0$ follows from (122). Now we also take into account the correction terms : Those relations among $(I^0 - XXIX^0)$ which are affected by L_{irr}^0 (107) are listed explicitly in App.C. On inspection one realizes that the choice (108) exactly cancels all terms $\sim \dot{\sigma}$ in $(I^0 - XXIX^0)$.

Thus all STI are fulfilled in our theory, and item ii) of the induction hypothesis is satisfied to loop order l. What remains to show is that the theory defined up to l-loop order is finite for $\Lambda_0 \to \infty$. As we noted, apart from the 9 evidently finite constants in (121), finiteness of R_1 and R_4 can be inferred from (125). To proceed further it is important to note that all irrelevant terms appearing in the STI apart from those in (107, 108) are a priori finite at l-loop since they only depend on the renormalization conditions at order $l' \leq l-1$. The next step is then to convince oneself of the fact that $(F^{AAA})^{0,\Lambda_0}$ has a finite limit for $\Lambda_0 \to \infty$. As we see from III_b the finiteness of F^{AAA} follows, if we can show that $\dot{\sigma}\,\delta m^2$ has a finite limit. From (122, 123, 124) it is evident that all relevant parameters fixed on the wrong side (at $\Lambda = \Lambda_0$) satisfy the bound assumed in the Corollary from Ch.3.2. From this Corollary (adapted to the Γ-functional) and the induction hypothesis we therefore conclude that δm^2 is bounded by $\Lambda_0^2\, \mathcal{P}(log\frac{\Lambda_0}{m})$, whereas $\dot{\sigma} \sim \Lambda_0^{-6}$. This proves the finiteness of F^{AAA}.[28] Then we go through the STI as follows:

$$F_1^{AAAA}\,(XIV_c),\; r_1^{AA\bar{c}c}\,(XIV_b),\; r_2^{AAAA}\,(XIV_a),\; r_2^{AA\bar{c}c}\,(XIV_e),\; r_1^{BB\bar{c}c}\,(XV_{1b}), \tag{134}$$
$$r^{\bar{c}c\bar{c}c}\,(XVIII_b)\,,\; r_2^{\bar{c}cA}\,(XVIII_c)\,,\; r_2^{AABB}\,(XXII)\,.$$

In parentheses we wrote the STI from which the finiteness of the respective relevant term may be inferred. In $r^{\bar{c}c\bar{c}c}\,(XVIII_b)$ note that $XVIII_b$ does not depend on $r_2^{\bar{c}cA}$ at l-loop order. We now infer from VII_d that

$$r_2^{hBA} \;=\; 2m\dot{\sigma}\,[F_0^{BBh}\,\frac{R_4^0}{R_1}]|_l \,+\, finite \;\text{ has a finite limit for }\; \Lambda_0 \to \infty\,. \tag{135}$$

Here the first contribution stems from the irrelevant term $\frac{m}{2}(2i_{20}^{\bar{c}ch} - i_{30}^{\bar{c}ch}) = -2m\,\dot{\sigma}\,F_0^{BBh}\,R_4^0$ in (108). In VII_d this contribution appears among the irrelevant terms and originates from the b.c. at $\Lambda = \Lambda_0$. Note that F_0^{BBh}, R_4^0 diverge at most linearly with Λ_0 using the results

[28] Using the STI we may in fact show at this stage that δm^2 diverges at most logarithmically.

from Ch.3.2 and Ch.4.2. Disposing of the finiteness of R_2 and r_2^{hBA} finiteness follows now also for
$R_6\,(XVI_a),\ R_7\,(XVIII_a),\ r^{hh\bar{c}c}\,(XVII_b),\ r^{hB\bar{c}c}\,(XXVIII),\ r_2^{BB\bar{c}c}\,(XXVII),\ F^{BBBB}\,(X)\,.$
Similarly as in (135) we may now conclude from V the finiteness of $F^{BBA}\,(V)\,.$ Next we pass through the following finiteness chain
$F_1^{\bar{c}cA}(IV_b),\ F_1^{AABB}(XV_{1a}),\ F^{AAh}(XIII_2), F_1^{hBA}(VI_b),\ F^{\bar{c}cB}(IV_a),\ \Sigma^{\bar{c}c}(VIII_a),\ \Sigma^{BB}(II_a),$
$\delta m^2(I_a),$ and then we can establish finiteness of $F^{\bar{c}ch}\,(VI_a)\,,\ R_5\,(VII_c$ or $XVI_b)\,.$
Finally it is easy to convince oneself of the finiteness of the remaining constants
$\Sigma^{hh}\,(VII_a)\,,\ \underline{\dot{\Sigma}}^{hh}\,(VII_b)\,,\ F^{BBhh}\,(XX)\,,\ F^{hhh}\,(IX)\,,\ F^{hhhh}\,(XIX)\,,\ F^{AAhh}\,(XVII_a)\,.$
In regarding the previous series of finiteness statements it is interesting to note that it is first extracted for the pure gauge sector and last for the terms involving the h field.[29] By now all of the 44 relevant constants are known to be finite, and thus item i) of the induction hypothesis to loop order l is verified.[30] Once i) ii) are verified, item iii) immediately follows from the general bounds in Ch.3.2 on noting that
a) from our choice of the bare action it is evident that $\partial^w \delta_\Phi^n L_1^0|_{0,0,l} \,=\, 0\,,\ |n|>5\,,$
b) the irrelevant terms in L_1^0 generated from those introduced in (107) on BRS transformation obey the required bound as a consequence of the previous finiteness statements
c) all other irrelevant terms in L_1^0 are generated by momentum derivatives acting on the regulating factor $\sigma_{0,\Lambda_0}(p)$, which automatically produces (more than) the required negative powers of Λ_0.

So the induction hypothesis holds to l-loop order. This ends the proof of the Theorem. ∎
Once the Theorem is proven, Proposition 4 tells us that the STI hold in the limit $\Lambda_0 \to \infty$.

Concluding Remarks

We have presented a renormalization proof for spontaneously broken Yang-Mills theory based on the Wilson renormalization group. The renormalization conditions admissible in view of the STI could be stated explicitly in (121) to (127).[31] We tried to avoid any equivocality as regards the analytical status of the statements we made, in particular for which values of the cutoffs they hold. We did not make use of unregularized path integrals. We think that the analytical aspect is generally somewhat neglected in the recent literature including textbooks. We did not attempt at generality on the symmetry or group theoretical aspects,

[29] This is reminiscent of the fact that the radiative corrections in the scalar boson sector are more rapidly divergent, namely quadratically, than all other ones.

[30] The smoothness assumption directly follows from the smoothness of the regulator and from the b.c. which depend on Λ_0 only through the regulator.

[31] Using in particular (123) it should be possible to derive the antighost equation of motion often used in textbooks [FaSl], [ZiJ].

which have been studied extensively in the literature, and restricted for simplicity to the physically interesting SU(2) case. We think it would be worth-while to extend the work - with the same precision on the analytical status - to the physical consequences to be drawn from the STI, in particular the gauge invariance of the S-Matrix. Further interesting problems to be treated in this context are the renormalization of QCD and the analysis of anomaly problems and of the action principle.

Appendix A

Here we consider the generating functional for the proper vertex functions

$$\Gamma(\underline{A}, \underline{h}, \underline{B}, \underline{\bar{c}}, \underline{c}) = \sum_{n=1}^{4} \Gamma_n + \Gamma_{(n>4)},$$

n counting the number of fields, and extract its relevant part, i.e. its local field content with mass dimension not greater than four. Generally we will not underline the field variable symbols in the Appendices, though of course all Γ functional arguments should be understood as such. In App.A and App.B the regulators are not explicited, apart from the subsequent comments on the two-point functions, where contributions arising for finite Λ_0 are explicited.
1) One-point function:

$$\Gamma_1 = \kappa \hat{h}(0).$$

2) Two-point functions:

$$\Gamma_2 = \int_p \left\{ \frac{1}{2} A^a_\mu(p) A^a_\nu(-p) \Gamma^{AA}_{\mu\nu}(p) + \frac{1}{2} h(p) h(-p) \Gamma^{hh}(p) + \frac{1}{2} B^a(p) B^a(-p) \Gamma^{BB}(p) \right.$$

$$\left. - \bar{c}^a(p) c^a(-p) \Gamma^{\bar{c}c}(p) + A^a_\mu(p) B^a(-p) \Gamma^{AB}_\mu(p) \right\},$$

$$\Gamma^{AA}_{\mu\nu}(p) = \delta_{\mu\nu}(m^2 + \delta m^2) + (p^2 \delta_{\mu\nu} - p_\mu p_\nu)(1 + \Sigma_{\text{trans}}(p^2)) + \frac{1}{\alpha} p_\mu p_\nu (1 + \Sigma_{\text{long}}(p^2)) \,,$$

$$\Gamma^{hh}(p) = p^2 + M^2 + \Sigma^{hh}(p^2) \,, \quad \Gamma^{BB}(p) = p^2 + \alpha m^2 + \Sigma^{BB}(p^2) \,,$$

$$\Gamma^{\bar{c}c}(p) = p^2 + \alpha m^2 + \Sigma^{\bar{c}c}(p^2) \,, \quad \Gamma^{AB}_\mu(p) = i p_\mu \Sigma^{AB}(p^2) \,.$$

Besides the unregularized tree order there emerge 10 relevant parameters from the various self energies: $\delta m^2, \Sigma_{\text{trans}}(0), \Sigma_{\text{long}}(0), \Sigma^{hh}(0), \dot{\Sigma}^{hh}(0), \Sigma^{BB}(0), \dot{\Sigma}^{BB}(0), \Sigma^{\bar{c}c}(0)$, $\dot{\Sigma}^{\bar{c}c}(0)$ and $\Sigma^{AB}(0)$, where we used the notation $\dot{\sum}(0) \equiv (\partial_{p^2} \sum)(0)$.
By (78, 79, 84) the 0-loop-order functional $\Gamma^{0,\Lambda_0}_{2,l=0}$ carries the inverted regulating factor

$(\sigma_{0,\Lambda_0})^{-1}(p^2) \;=\; 1 - \dot\sigma\, p^2 + \mathcal{O}((p^2)^2)$ with $\dot\sigma \;=\; -(\alpha m^4 + (1+\alpha)m^2M^2)/\Lambda_0^6$. Therefore all self energies vanish at order $l=0$, whereas

$$\dot\Sigma^{hh}_{l=0}(0) \;=\; -\dot\sigma\, M^2\,, \quad \dot\Sigma^{BB}_{l=0}(0) \;=\; \dot\Sigma^{\bar c c}_{l=0}(0) \;=\; -\dot\sigma\,\alpha\, m^2\,, \tag{136}$$

$$\Sigma_{trans}|_{l=0}(0) \;=\; -\dot\sigma\, m^2\,, \quad \Sigma_{long}|_{l=0}(0) \;=\; -\dot\sigma\,\alpha\, m^2\,.$$

To clearly isolate the tree level cutoff effects from the loop contributions we introduce the notation

$$\underline{\Sigma}(0) \;=\; \Sigma(0) \;-\; \Sigma(0)|_{l=0}\,, \quad \underline{\dot\Sigma}(0) \;=\; \dot\Sigma(0) \;-\; \dot\Sigma(0)|_{l=0}\,. \tag{137}$$

3) Three-point functions:

Only the relevant part is given explicitly: $r = \mathcal{O}(\hbar)$ denotes a relevant parameter which vanishes in the tree order, otherwise a relevant parameter is denoted by F. Moreover, we indicate an irrelevant part by a symbol $\mathcal{O}_n,\ n\in\mathbb{N}$, indicating that this part vanishes as an n-th power of the momentum in the limit when all momenta tend to zero homogeneously.

$$\begin{aligned}\Gamma_3 \;=\; \int_p\int_q \Big\{ & \epsilon^{rst}A^r_\mu(p)A^s_\nu(q)A^t_\lambda(-p-q)\Gamma^{AAA}_{\mu\nu\lambda}(p,q)\\ & +A^r_\mu(p)A^r_\nu(q)h(-p-q)\Gamma^{AAh}_{\mu\nu}(p,q) + \epsilon^{rst}B^r(p)B^s(q)A^t_\mu(-p-q)\Gamma^{BBA}_\mu(p,q)\\ & +h(p)B^r(q)A^r_\mu(-p-q)\Gamma^{hBA}_\mu(p,q) + \epsilon^{rst}\bar c^r(p)c^s(q)A^t_\mu(-p-q)\Gamma^{\bar c cA}_\mu(p,q)\\ & +B^r(p)B^r(q)h(-p-q)\Gamma^{BBh}(p,q) + h(p)h(q)h(-p-q)\Gamma^{hhh}(p,q)\\ & +\bar c^r(p)c^r(q)h(-p-q)\Gamma^{\bar cch}(p,q) + \epsilon^{rst}\bar c^r(p)c^s(q)B^t(-p-q)\Gamma^{\bar ccB}(p,q)\Big\}\,,\end{aligned}$$

$$\begin{aligned}
\Gamma^{AAA}_{\mu\nu\lambda}(p,q) &= \delta_{\mu\nu} i(p-q)_\lambda F^{AAA} + \mathcal{O}_3, & F^{AAA} &= -\tfrac{1}{2}g + r^{AAA},\\
\Gamma^{AAh}_{\mu\nu}(p,q) &= \delta_{\mu\nu}F^{AAh} + \mathcal{O}_2, & F^{AAh} &= \tfrac{1}{2}mg + r^{AAh},\\
\Gamma^{BBA}_{\mu}(p,q) &= i(p-q)_\mu F^{BBA} + \mathcal{O}_3, & F^{BBA} &= -\tfrac{1}{4}g + r^{BBA},\\
\Gamma^{hBA}_{\mu}(p,q) &= i(p-q)_\mu F^{hBA}_1 & F^{hBA}_1 &= \tfrac{1}{2}g + r^{hBA}_1,\\
&\quad +i(p+q)_\mu r^{hBA}_2 + \mathcal{O}_3, & &\\
\Gamma^{\bar ccA}_{\mu}(p,q) &= ip_\mu F^{\bar ccA}_1 + iq_\mu r^{\bar ccA}_2 + \mathcal{O}_3, & F^{\bar ccA}_1 &= g + r^{\bar ccA}_1,\\
\Gamma^{BBh}(p,q) &= F^{BBh} + \mathcal{O}_2, & F^{BBh} &= \tfrac{1}{4}g\tfrac{M^2}{m} + r^{BBh},\\
\Gamma^{hhh}(p,q) &= F^{hhh} + \mathcal{O}_2, & F^{hhh} &= \tfrac{1}{4}g\tfrac{M^2}{m} + r^{hhh},\\
\Gamma^{\bar cch}(p,q) &= F^{\bar cch} + \mathcal{O}_2, & F^{\bar cch} &= -\tfrac{1}{2}\alpha gm + r^{\bar cch},\\
\Gamma^{\bar ccB}(p,q) &= F^{\bar ccB} + \mathcal{O}_2, & F^{\bar ccB} &= \tfrac{1}{2}\alpha gm + r^{\bar ccB}.
\end{aligned}$$

The 3-point functions AAB and BBB have no relevant local content.

4) Four-point functions: With parameters r and F defined as before

$$
\begin{aligned}
\Gamma_4|_{\rm rel} \;=\; & \int_k\int_p\int_q \Big\{ \epsilon^{abc}\epsilon^{ars} A_\mu^b(k)A_\nu^c(p)A_\mu^r(q)A_\nu^s(-k-p-q)F_1^{AAAA} \\
& +A_\mu^r(k)A_\mu^r(p)A_\nu^s(q)A_\nu^s(-k-p-q)r_2^{AAAA} \\
& +A_\mu^a(k)A_\mu^b(p)\bar{c}^r(q)c^s(-k-p-q)(\delta^{ab}\delta^{rs}r_1^{AA\bar{c}c}+\delta^{ar}\delta^{bs}r_2^{AA\bar{c}c}) \\
& +A_\mu^a(k)A_\mu^b(p)B^r(q)B^s(-k-p-q)(\delta^{ab}\delta^{rs}F_1^{AABB}+\delta^{ar}\delta^{bs}r_2^{AABB}) \\
& +B^a(k)B^b(p)\bar{c}^r(q)c^s(-k-p-q)(\delta^{ab}\delta^{rs}r_1^{BB\bar{c}c}+\delta^{ar}\delta^{bs}r_2^{BB\bar{c}c}) \\
& +h(k)h(p)h(q)h(-k-p-q)F^{hhhh} \\
& +B^r(k)B^r(p)h(q)h(-k-p-q)F^{BBhh} \\
& +B^r(k)B^r(p)B^s(q)B^s(-k-p-q)F^{BBBB} \\
& +A_\mu^r(k)A_\mu^r(p)h(q)h(-k-p-q)F^{AAhh} \\
& +h(k)h(p)\bar{c}^r(q)c^r(-k-p-q)r^{hh\bar{c}c} \\
& +\bar{c}^a(k)c^a(p)\bar{c}^r(q)c^r(-k-p-q)r^{\bar{c}c\bar{c}c} \\
& +\epsilon^{rst}h(k)B^r(p)\bar{c}^s(q)c^t(-k-p-q)r^{hB\bar{c}c}\Big\},
\end{aligned}
$$

$$
\begin{aligned}
F_1^{AAAA} &= \tfrac{1}{4}g^2+r_1^{AAAA}, & F_1^{AABB} &= \tfrac{1}{8}g^2+r_1^{AABB}, \\
F^{hhhh} &= \tfrac{1}{32}g^2\left(\tfrac{M}{m}\right)^2+r^{hhhh}, & F^{BBhh} &= \tfrac{1}{16}g^2\left(\tfrac{M}{m}\right)^2+r^{BBhh}, \\
F^{BBBB} &= \tfrac{1}{32}g^2\left(\tfrac{M}{m}\right)^2+r^{BBBB}, & F^{AAhh} &= \tfrac{1}{8}g^2+r^{AAhh}.
\end{aligned}
$$

Hence, in total Γ involves $1+10+11+15=37$ relevant parameters.

Appendix B

We also have to consider the vertex functions with operator insertions stemming from the BRS-transforms. These insertions have mass dimension ≤ 2.
Only the respective relevant part of the four vertex functions with insertions is listed:

$$
\begin{aligned}
\Gamma_{\gamma_\mu^a(p)}|_{\rm rel} &= -ip_\mu c^a(-p)R_1+\epsilon^{arb}\int_q A_\mu^r(q)c^b(-p-q)gR_2, \\
\Gamma_{\gamma(p)}|_{\rm rel} &= \int_q B^r(q)c^r(-p-q)(-\frac{1}{2}gR_3),
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_{\gamma^a(p)}|_{\rm rel} &= mc^a(-p)R_4 + \int_q h(q)c^a(-p-q)\frac{1}{2}gR_5 + \epsilon^{arb}\int_q B^r(q)c^b(-p-q)\frac{1}{2}gR_6,\\
\Gamma_{\omega^a(p)}|_{\rm rel} &= \epsilon^{ars}\int_q c^r(q)c^s(-p-q)\frac{1}{2}gR_7.
\end{aligned}
$$

There appear 7 relevant parameters

$$
R_i = 1 + r_i, \quad r_i = \mathcal{O}(\hbar), \quad i = 1, ..., 7.
$$

All other 2-point functions, and the higher ones, of course, are of irrelevant type.

Appendix C

Here we present the 53 conditions which result upon requiring that the functional Γ_1, (83), has a vanishing local part for (mass) dimensions smaller or equal to five

$$
\Gamma_1(\underline{A}, \underline{h}, \underline{B}, \underline{\bar{c}}, \underline{c})|_{\dim\leq 5} \stackrel{!}{=} 0.
$$

Into most of these conditions also irrelevant contributions enter which are not given explicitly but are simply indicated by "irr". To recognize the local origin, we keep the momentum factors arising. The δ-distribution emerging from the functional derivatives and forcing the sum of the corresponding momenta to zero is not written. Relations explicitly rewritten for L^0 carry a zero in the numbering. In those, the irrelevant terms from (107,108) are the only ones appearing and are written explicitly.

The STI for Γ are supposed to be written for the case $\Lambda = 0$, $\Lambda_0 \leq \infty$. Note that they take different form for $\Lambda_0 < \infty$ and $\Lambda_0 \to \infty$ only, if $\dot{\sigma}$ appears, which is the case in I_b, II_b, III_b, V, VII_b, VII_c, VII_d, $VIII_b$, $VIII_c$. For the L^0-functional we write for the loop level two-point functions Σ_0 instead of Σ and $\underline{\Sigma}_0$, $\dot{\underline{\Sigma}}_0$ instead of $\underline{\Sigma}$, $\dot{\underline{\Sigma}}$.

Two fields

I) $\delta_{A^a_\mu(q)}\delta_{c^r(k)}\Gamma_1|_0$

a) $0 \stackrel{!}{=} q_\mu\left\{-(m^2+\delta m^2)R_1 + \Sigma^{AB}(0)mR_4 + m^2 + \frac{1}{\alpha}\Sigma^{\bar{c}c}(0)\right\}$,

b) $0 \stackrel{!}{=} q^2 q_\mu\left\{-\frac{1}{\alpha}(1+\underline{\Sigma}_{\rm long}(0))R_1 + \frac{1}{\alpha}(1+\dot{\underline{\Sigma}}^{\bar{c}c}(0)) - \dot{\sigma}[\delta m^2 R_1 - \Sigma^{AB}(0)mR_4 - \frac{1}{\alpha}\Sigma^{\bar{c}c}(0)] + {\rm irr}\right\}$.

II) $\delta_{B^a(q)}\delta_{c^r(k)}\Gamma_1|_0$

a) $0 \stackrel{!}{=} m(\alpha m^2 + \Sigma^{BB}(0))R_4 - m(\alpha m^2 + \Sigma^{\bar{c}c}(0)) + \kappa(-\frac{1}{2}g)R_3$,

b) $0 \stackrel{!}{=} q^2\Big\{-\sum^{AB}(0)R_1+m(1+\dot{\underline{\Sigma}}^{BB}(0))R_4-m(1+\dot{\underline{\Sigma}}^{\bar{c}c}(0))-\dot{\sigma}m[\Sigma^{\bar{c}c}(0)-\Sigma^{BB}(0)R_4]+\mathrm{irr}\Big\}.$

Three fields

III) $\delta_{A^r_\mu(p)}\delta_{A^s_\nu(q)}\delta_{c^t(k)}\Gamma_1|_0$

a) $0 \stackrel{!}{=} (p_\mu p_\nu - q_\mu q_\nu)\Big\{-2F^{AAA}R_1-\frac{1}{\alpha}(F_1^{\bar{c}cA}-r_2^{\bar{c}cA})+\Big[\frac{1}{\alpha}(1+\underline{\Sigma}_{\mathrm{long}}(0))-(1+\underline{\Sigma}_{\mathrm{trans}}(0))\Big]\,gR_2+\mathrm{irr}\Big\},$

b) $0 \stackrel{!}{=} (p^2-q^2)\delta_{\mu\nu}\Big\{2F^{AAA}R_1+(1+\underline{\Sigma}_{\mathrm{trans}}(0))gR_2+\dot{\sigma}\,\delta m^2 gR_2+\mathrm{irr}\Big\},$

b^0) $0 \stackrel{!}{=} (p^2-q^2)\delta_{\mu\nu}\Big\{2F_0^{AAA}R_1^0+(1+\underline{\Sigma}_{0,\mathrm{trans}}(0))gR_2^0+\dot{\sigma}\,\delta m_0^2\,gR_2^0+2\,i_{10}^{AAB}m\,R_4^0\Big\},$

IV) $\delta_{A^r_\mu(p)}\delta_{B^s(q)}\delta_{c^t(k)}\Gamma_1|_0$

a) $0 \stackrel{!}{=} p_\mu\Big\{2F^{BBA}mR_4+\frac{1}{2}g\sum^{AB}(0)R_6+\frac{1}{\alpha}F^{\bar{c}cB}-mr_2^{\bar{c}cA}+\mathrm{irr}\Big\},$

b) $0 \stackrel{!}{=} q_\mu\Big\{g\sum^{AB}(0)R_2+4F^{BBA}mR_4+m(F_1^{\bar{c}cA}-r_2^{\bar{c}cA})+\mathrm{irr}\Big\},$

V) $\delta_{B^r(p)}\delta_{B^s(q)}\delta_{c^t(k)}\Gamma_1|_0$

$0 \stackrel{!}{=} (p^2-q^2)\Big\{2R_1F^{BBA}+(1+\dot{\underline{\Sigma}}^{BB}(0))\frac{g}{2}R_6-\dot{\sigma}[m\,F^{\bar{c}cB}-\Sigma^{BB}(0)\frac{g}{2}R_6]+\mathrm{irr}\Big\},$

V^0) $0 \stackrel{!}{=} (p^2-q^2)\Big\{2R_1^0F_0^{BBA}+(1+\dot{\underline{\Sigma}}_0^{BB}(0))\frac{g}{2}R_6^0-\dot{\sigma}[m\,F_0^{\bar{c}cB}-\Sigma_0^{BB}(0)\frac{g}{2}R_6^0]-m(i_{10}^{\bar{c}cB}-i_{30}^{\bar{c}cB})\Big\},$

VI) $\delta_{A^r_\mu(p)}\delta_{h(q)}\delta_{c^t(k)}\Gamma_1|_0$

a) $0 \stackrel{!}{=} p_\mu\Big\{-2R_1F^{AAh}+mR_4(F_1^{hBA}-r_2^{hBA})+\sum^{AB}(0)\frac{1}{2}gR_5-\frac{1}{\alpha}F^{\bar{c}ch}+\mathrm{irr}\Big\},$

b) $0 \stackrel{!}{=} q_\mu\Big\{-2R_1F^{AAh}+2mR_4F_1^{hBA}+\mathrm{irr}\Big\},$

VII) $\delta_{h(p)}\delta_{B^s(q)}\delta_{c^t(k)}\Gamma_1|_0$

a) $0 \stackrel{!}{=} (M^2+\sum^{hh}(0))(-\frac{1}{2}gR_3)+2mF^{BBh}R_4+mF^{\bar{c}ch}+(\alpha m^2+\sum^{BB}(0))\frac{1}{2}gR_5,$

b) $0 \stackrel{!}{=} p^2\Big\{F_1^{hBA}R_1-(1+\dot{\underline{\Sigma}}^{hh}(0))\frac{1}{2}gR_3-\dot{\sigma}\sum^{hh}(0)\frac{1}{2}gR_3+\mathrm{irr}\Big\},$

b^0) $0 \stackrel{!}{=} p^2\Big\{F_1^{hBA}R_1^0-(1+\dot{\underline{\Sigma}}_0^{hh}(0))\frac{1}{2}gR_3^0-\dot{\sigma}\sum_0^{hh}(0)\frac{1}{2}gR_3^0+\frac{1}{2}\,m\,i_{30}^{\bar{c}ch}\Big\},$

c) $0 \stackrel{!}{=} q^2\Big\{-F_1^{hBA}R_1+(1+\dot{\underline{\Sigma}}^{BB}(0))\frac{1}{2}gR_5+\dot{\sigma}[mF^{\bar{c}ch}+\sum^{BB}(0)\frac{1}{2}gR_5]+\mathrm{irr}\Big\},$

c^0) $0 \stackrel{!}{=} q^2\Big\{-F_{10}^{hBA}R_1^0+(1+\dot{\underline{\Sigma}}_0^{BB}(0))\frac{1}{2}gR_5^0+\dot{\sigma}[mF_0^{\bar{c}ch}+\sum_0^{BB}(0)\frac{1}{2}gR_5^0]+\frac{1}{2}\,m\,(2i_{10}^{\bar{c}ch}-i_{30}^{\bar{c}ch})\Big\},$

d) $0 \stackrel{!}{=} k^2\Big\{r_2^{hBA}R_1+\dot{\sigma}\,2mF^{BBh}R_4+\mathrm{irr}\Big\},$

d^0) $0 \stackrel{!}{=} k^2\Big\{r_{20}^{hBA}R_1^0+\dot{\sigma}\,2mF_0^{BBh}R_4^0+\frac{1}{2}\,m\,(2i_{20}^{\bar{c}ch}-i_{30}^{\bar{c}ch})\Big\},$

VIII) $\delta_{c^t(q)}\delta_{c^s(p)}\delta_{\bar{c}^r(k)}\Gamma_1|_0$

$$
\begin{aligned}
\text{a)}\ 0 &\stackrel{!}{=} 2mF^{\bar{c}cB}R_4 - (\alpha m^2 + \textstyle\sum^{\bar{c}c}(0))gR_7,\\
\text{b)}\ 0 &\stackrel{!}{=} k^2\Big\{F_1^{\bar{c}cA}R_1 - r_2^{\bar{c}cA}R_1 - (1+\dot{\underline{\Sigma}}^{\bar{c}c}(0))gR_7 - \dot{\sigma}\ \textstyle\sum^{\bar{c}c}(0))gR_7 + \mathrm{irr}\Big\},\\
b^0)\ 0 &\stackrel{!}{=} k^2\Big\{F_{10}^{\bar{c}cA}R_1^0 - r_{20}^{\bar{c}cA}R_1^0 - (1+\dot{\underline{\Sigma}}_0^{\bar{c}c}(0))gR_7^0 - \dot{\sigma}\textstyle\sum_0^{\bar{c}c}(0))gR_7^0 + m\,R_4^0(2i_{10}^{\bar{c}cB} - i_{30}^{\bar{c}cB})\Big\},\\
\text{c)}\ 0 &\stackrel{!}{=} (p^2+q^2)\Big\{r_2^{\bar{c}cA}R_1 + \dot{\sigma}\,mF^{\bar{c}cB}R_4 + \mathrm{irr}\Big\}.\\
c^0)\ 0 &\stackrel{!}{=} (p^2+q^2)\Big\{r_{20}^{\bar{c}cA}R_1^0 + \dot{\sigma}\,mF_0^{\bar{c}cB}R_4^0 + m\,R_4^0\,i_{20}^{\bar{c}cB}\Big\}.
\end{aligned}
$$

Four fields

IX) $\delta_{h(p)}\delta_{h(q)}\delta_{B^1(k)}\delta_{c^1(l)}\Gamma_1|_0$

$$0 \stackrel{!}{=} 6F^{hhh}(-\tfrac{1}{2}gR_3) + 4F^{BBhh}mR_4 + 2F^{BBh}gR_5 + 2mr^{hh\bar{c}c} + \mathrm{irr}.$$

X) $\delta_{B^1(k)}\delta_{B^1(p)}\delta_{B^2(q)}\delta_{c^2(l)}\Gamma_1|_0$

$$0 \stackrel{!}{=} -F^{BBh}gR_3 + 8F^{BBBB}mR_4 + m\left(2r_1^{BB\bar{c}c} + r_2^{BB\bar{c}c}\right) + \mathrm{irr}.$$

XI) $\delta_{h(l)}\delta_{\bar{c}^3(k)}\delta_{c^1(p)}\delta_{c^2(q)}\Gamma_1|_0$

$$0 \stackrel{!}{=} 2r^{hB\bar{c}c}mR_4 + F^{\bar{c}cB}gR_5 + F^{\bar{c}ch}gR_7 + \mathrm{irr}.$$

XII) $\delta_{c^2(k)}\delta_{\bar{c}^2(l)}\delta_{c^1(p)}\delta_{B^1(q)}\Gamma_1|_0$

$$0 \stackrel{!}{=} F^{\bar{c}ch}(-\tfrac{1}{2}gR_3) + (2r_1^{BB\bar{c}c} - r_2^{BB\bar{c}c})mR_4 + F^{\bar{c}cB}(\tfrac{1}{2}gR_6 - gR_7) + 2mr^{\bar{c}c\bar{c}c} + \mathrm{irr}.$$

XIII)$_1$ $\delta_{A_\mu^1(k)}\delta_{A_\nu^2(p)}\delta_{B^1(q)}\delta_{c^2(l)}\Gamma_1|_0$

$$0 \stackrel{!}{=} 2r_2^{AABB}R_4 + r_2^{AA\bar{c}c} + \mathrm{irr}.$$

XIII)$_2$ $\delta_{A_\mu^1(k)}\delta_{A_\nu^1(p)}\delta_{B^2(q)}\delta_{c^2(l)}\Gamma_1|_0$

$$0 \stackrel{!}{=} -F^{AAh}gR_3 + 4F_1^{AABB}mR_4 + 2mr_1^{AA\bar{c}c} + \mathrm{irr}.$$

XIV) $\delta_{A_\mu^1(p)}\delta_{A_\nu^1(q)}\delta_{A_\rho^2(k)}\delta_{c^2(l)}\Gamma_1|_0$

$$
\begin{aligned}
\text{a)}\ 0 &\stackrel{!}{=} 2\delta_{\mu\nu}l_\rho\Big\{4(F_1^{AAAA} + r_2^{AAAA})R_1 + 2F^{AAA}gR_2 + \tfrac{1}{\alpha}r_1^{AA\bar{c}c} + \mathrm{irr}\Big\},\\
\text{b)}\ 0 &\stackrel{!}{=} \delta_{\mu\nu}(p_\rho + q_\rho)\left\{\tfrac{2}{\alpha}r_1^{AA\bar{c}c} + \mathrm{irr}\right\},
\end{aligned}
$$

c) $0 \stackrel{!}{=} (\delta_{\mu\rho} l_\nu + \delta_{\nu\rho} l_\mu) \Big\{ -4F_1^{AAAA} R_1 - 2F^{AAA} g R_2 + \mathrm{irr} \Big\},$

d) $0 \stackrel{!}{=} (\delta_{\mu\rho} p_\nu + \delta_{\nu\rho} q_\mu) \{ 0 + \mathrm{irr} \},$

e) $0 \stackrel{!}{=} (\delta_{\mu\rho} q_\nu + \delta_{\nu\rho} p_\mu) \Big\{ -\frac{1}{\alpha} r_2^{AA\bar{c}c} + \mathrm{irr} \Big\}.$

XV)$_1$ $\delta_{B^1(p)} \delta_{B^1(q)} \delta_{A_\mu^2(k)} \delta_{c^2(l)} \Gamma_1 |_0$

a) $0 \stackrel{!}{=} l_\mu \Big\{ 4F_1^{AABB} R_1 + 2F^{BBA} g R_6 + \mathrm{irr} \Big\},$

b) $0 \stackrel{!}{=} k_\mu \Big\{ r_1^{BB\bar{c}c} + \mathrm{irr} \Big\},$

XV)$_2$ $\delta_{B^1(p)} \delta_{B^2(q)} \delta_{A_\mu^1(k)} \delta_{c^2(l)} \Gamma_1 |_0$

a) $0 \stackrel{!}{=} p_\mu \Big\{ -2r_2^{AABB} R_1 + 2F^{BBA} g R_2 + F_1^{hBA} g R_3 + \mathrm{irr} \Big\},$

b) $0 \stackrel{!}{=} q_\mu \Big\{ -2r_2^{AABB} R_1 - 2F^{BBA} g R_2 + 2F^{BBA} g R_6 + \mathrm{irr} \Big\},$

c) $0 \stackrel{!}{=} k_\mu \Big\{ -2r_2^{AABB} R_1 + F_1^{hBA} \frac{1}{2} g R_3 + r_2^{hBA} \frac{1}{2} g R_3 + F^{BBA} g R_6 - \frac{1}{\alpha} r_2^{BB\bar{c}c} + \mathrm{irr} \Big\},$

XVI) $\delta_{h(p)} \delta_{A_\mu^1(k)} \delta_{B^2(q)} \delta_{c^3(l)} \Gamma_1 |_0$

a) $0 \stackrel{!}{=} p_\mu \Big\{ F_1^{hBA} g (R_6 - R_2) - r_2^{hBA} g R_2 + \mathrm{irr} \Big\},$

b) $0 \stackrel{!}{=} q_\mu \Big\{ F_1^{hBA} g R_2 - r_2^{hBA} g R_2 + 2F^{BBA} g R_5 + \mathrm{irr} \Big\},$

c) $0 \stackrel{!}{=} k_\mu \Big\{ F_1^{hBA} \frac{1}{2} g R_6 - r_2^{hBA} \frac{1}{2} g R_6 + F^{BBA} g R_5 - \frac{1}{\alpha} r^{hB\bar{c}c} + \mathrm{irr} \Big\},$

XVII) $\delta_{h(p)} \delta_{h(q)} \delta_{A_\mu^1(k)} \delta_{c^1(l)} \Gamma_1 |_0$

a) $0 \stackrel{!}{=} l_\mu \Big\{ 4F^{AAhh} R_1 - F_1^{hBA} g R_5 + \mathrm{irr} \Big\},$

b) $0 \stackrel{!}{=} k_\mu \Big\{ r_2^{hBA} g R_5 + \frac{2}{\alpha} r^{hh\bar{c}c} + \mathrm{irr} \Big\}.$

XVIII) $\delta_{A_\mu^2(k)} \delta_{c^2(p)} \delta_{c^1(q)} \delta_{\bar{c}^1(l)} \Gamma_1 |_0$

a) $0 \stackrel{!}{=} l_\mu \Big\{ F_1^{\bar{c}cA} g (R_2 - R_7) + \frac{2}{\alpha} r^{\bar{c}c\bar{c}c} + \mathrm{irr} \Big\},$

b) $0 \stackrel{!}{=} p_\mu \Big\{ 2r_1^{AA\bar{c}c} R_1 + r_2^{\bar{c}cA} g (R_2 - R_7) + \frac{2}{\alpha} r^{\bar{c}c\bar{c}c} + \mathrm{irr} \Big\},$

c) $0 \stackrel{!}{=} q_\mu \Big\{ -r_2^{AA\bar{c}c} R_1 - r_2^{\bar{c}cA} g R_7 + \frac{2}{\alpha} r^{\bar{c}c\bar{c}c} + \mathrm{irr} \Big\}.$

Five fields

XIX) $\delta_{h(p)} \delta_{h(q)} \delta_{h(k)} \delta_{B^1(l)} \delta_{c^1(l')} \Gamma_1 |_0$

$$0 \stackrel{!}{=} -2F^{hhhh} R_3 + F^{hhBB} R_5 + \mathrm{irr}.$$

XX) $\delta_{h(p)}\delta_{B^1(q)}\delta_{B^1(k)}\delta_{B^2(l)}\delta_{c^2(l')}\Gamma_1|_0$

$$0 \stackrel{!}{=} -F^{BBhh}R_3 + 2F^{BBBB}R_5 + \mathrm{irr.}$$

XXI) $\delta_{A^1_\mu(k)}\delta_{A^1_\nu(p)}\delta_{h(k)}\delta_{B^2(l)}\delta_{c^2(l')}\Gamma_1|_0$

$$0 \stackrel{!}{=} -F^{AAhh}R_3 + F_1^{AABB}R_5 + \mathrm{irr.}$$

XXII) $\delta_{A^1_\mu(k)}\delta_{B^1(p)}\delta_{c^1(l')}\delta_{A^2_\nu(q)}\delta_{B^3(l)}\Gamma_1|_0$

$$0 \stackrel{!}{=} r_2^{AABB}(R_6 - 2R_2) + \mathrm{irr.}$$

XXIII) $\delta_{A^1_\mu(k)}\delta_{B^1(q)}\delta_{A^2_\nu(p)}\delta_{c^2(l')}\delta_{h(l)}\Gamma_1|_0$

$$0 \stackrel{!}{=} r_2^{AABB}R_5 + \mathrm{irr.}$$

XXIV) $\delta_{A^3_\mu(k)}\delta_{A^3_\nu(p)}\delta_{\bar{c}^2(q)}\delta_{c^3(l)}\delta_{c^1(l')}\Gamma_1|_0$

$$0 \stackrel{!}{=} r_2^{AA\bar{c}c}R_2 + r_1^{AA\bar{c}c}R_7 + \mathrm{irr.}$$

XXV) $\delta_{A^3_\mu(k)}\delta_{\bar{c}^3(q)}\delta_{A^2_\nu(p)}\delta_{c^3(l)}\delta_{c^1(l')}\Gamma_1|_0$

$$0 \stackrel{!}{=} r_2^{AA\bar{c}c}(3R_2 - R_7) + \mathrm{irr.}$$

XXVI) $\delta_{B^1(p)}\delta_{B^1(q)}\delta_{\bar{c}^1(k)}\delta_{c^2(l)}\delta_{c^3(l')}\Gamma_1|_0$

$$0 \stackrel{!}{=} r_2^{BB\bar{c}c}(R_6 - R_7) - r_1^{BB\bar{c}c}R_7 + \mathrm{irr.}$$

XXVII) $\delta_{B^1(p)}\delta_{\bar{c}^1(k)}\delta_{B^2(q)}\delta_{c^3(l)}\delta_{c^1(l')}\Gamma_1|_0$

$$0 \stackrel{!}{=} -r^{hB\bar{c}c}R_3 + r_2^{BB\bar{c}c}(3R_6 - 2R_7) + \mathrm{irr.}$$

XXVIII) $\delta_{h(p)}\delta_{h(q)}\delta_{\bar{c}^1(k)}\delta_{c^2(l)}\delta_{c^3(l')}\Gamma_1|_0$

$$0 \stackrel{!}{=} r^{hB\bar{c}c}R_5 + r^{hh\bar{c}c}R_7 + \mathrm{irr.}$$

XXIX) $\delta_{h(p)}\delta_{B^1(q)}\delta_{c^1(l)}\delta_{\bar{c}^2(k)}\delta_{c^2(l')}\Gamma_1|_0$

$$0 \stackrel{!}{=} 2r^{hh\bar{c}c}R_3 - 2r_1^{BB\bar{c}c}R_5 + r_2^{BB\bar{c}c}R_5 + r^{hB\bar{c}c}(-R_6 + 2R_7) + \mathrm{irr.}$$

These 53 conditions are fulfilled in the (tree) order $\hbar^0$ for $\Lambda = 0$ and $\Lambda_0 \leq \infty$. For finite Λ_0 we have also to take into account the tree order irrelevant contribution from (107, 108) to the classical action.

References:

[BAM1] M.Bonini, M.D'Attanasio, G.Marchesini: Ward identities and Wilson renormalization group for QED. Nucl.Phys. **B418**, 81-112 (1994)

[BAM2] M.Bonini, M.D'Attanasio, G.Marchesini: BRS symmetry for Yang-Mills theory with exact renormalization group. Nucl.Phys. **B437**, 163-186 (1995)

[Ell] U.Ellwanger: Flow equations and Bound states in Quantum Field theory. Zeitsch.f. Physik **C38**, 619-629 (1993)

[EHW] U.Ellwanger, M.Hirsch, A.Weber, Flow Equations for the relevant part of the pure Yang-Mills action. Zeitsch.f.Physik **C69**, 687-697 (1996)

[FaSl] L.D.Faddeev, A.A.Slavnov: Gauge Fields: Introduction to Quantum Theory. Benjamin, Reading (Mass.), 1980

[Ke1] G.Keller: The Perturbative Construction of Symanzik's improved Action for ϕ_4^4 and QED_4. Helv.Phys.Acta **66**, 453 (1993)

[Ke2] G.Keller: Local Borel summability of Euclidean Φ_4^4: A simple Proof via Differential Flow Equations. Commun.Math.Phys. **161**, 311-323 (1994)

[Kim] C.Kim: A Renormalization Group Flow Approach to Decoupling and Irrelevant Operators. Ann.Phys.(N.Y.) **243**, 117-143(1995)

[KK1] G.Keller, Ch.Kopper: Perturbative Renormalization of Massless Φ_4^4 with Flow Equations. Commun.Math.Phys. **161**, 515-532 (1994)

[KK2] G.Keller, Ch.Kopper: Perturbative Renormalization of QED via flow equations. Phys. Lett. **B273**, 323-332 (1991)
Renormalizability Proof for QED Based on Flow Equations. Commun.Math.Phys. **176**, 193-226 (1996)

[KK3] G.Keller, Ch.Kopper: Perturbative Renormalization of Composite Operators via Flow Equations I. Commun.Math.Phys. **148**, 445-467 (1992)

[KK4] G.Keller, Ch.Kopper: Perturbative Renormalization of Composite Operators via Flow Equations II: Short distance expansion. Commun.Math.Phys. **153**, 245-276 (1993)

[KKSa] G.Keller, Ch.Kopper, M.Salmhofer: Perturbative Renormalization and Effective Lagrangians in Φ_4^4. Helv.Phys.Acta **65**, 32-52 (1991)

[KKSc] G.Keller, Ch.Kopper, C.Schophaus: Perturbative Renormalization with Flow Equations in Minkowski Space. Helv.Phys.Acta **70**, 247-274 (1997)

[Kop] Ch.Kopper: Renormierungstheorie mit Flußgleichungen. Shaker Verlag, Aachen, 1998

[Pol] J. Polchinski: Renormalization and Effective Lagrangians. Nucl.Phys.**B231**, 269-295 (1984)

[Rei] Th.Reiß: Lattice Gauge Theory: Renormalization to all orders in the Loop Expansion. Nucl.Phys. **B313**, 417-463(1989), and previous work of this author cited there.

[ReWe] M.Reuter, Ch.Wetterich: Gluon Condensation in Nonperturbative Flow Equations. Phys.Rev. **D56**, 7893-7916 (1997)

[TeWe] N.Tetradis, Ch.Wetterich: Critical exponents from the Average Action. Nucl.Phys. B422, 541-592 (1994)

[Wet] Ch.Wetterich: Exact evolution equation for the effective potential. Phys.Lett.**B301**, 90-94 (1993)

[Wie] Ch.Wieczerkowski: Symanzik's Improved actions from the viewpoint of the Renormalization Group. Commun.Math.Phys. **120**, 148-176 (1988)

[WiKo] K.Wilson, J.B.Kogut: The Renormalization Group and the ε-Expansion. Phys.Rep. **12C**, 75-199 (1974)

[ZiJ] J.Zinn-Justin: Quantum Field Theory and Critical Phenomena, Clarendon Press,Oxford, 3rd ed. 1997