The Renormalization-Group peculiarities of Griffiths and Pearce: What have we learned?

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Abstract:
We review what we have learned about the "Renormalization Group peculiarities" which were discovered about twenty years ago by Griffiths and Pearce, and which questions they asked are still widely open. We also mention some related developments.

Keywords: Renormalization-Group peculiarities, non-Gibbsian measures.

1 Introduction

About twenty years ago, Griffiths and Pearce [23, 24] discovered some unexpected mathematical difficulties in rigorously implementing many of the generally used real-space Renormalization Group transformations.

In this contribution I plan to assess what we have learned about these problems since then. In particular we will see that, although it turns out that Renormalization-Group maps cannot be discontinuous, they can be ill-defined. This means that in such cases no "reasonable" renormalized Hamiltonian can be found.

Moreover, in any region of the phase diagram the question whether a particular transformation is well-defined or ill-defined turns out to be highly non-trivial. The ill-definedness of various Renormalization-Group maps can be expressed in the violation of the property of "quasilocality" in the renormalized states. The study and classification of such non-quasilocal states (—non-Gibbsian measures—) has led also to various results of mathematical interest, some of which we will mention further on. Some other papers covering the area of non-Gibbsianness and Renormalization-Group peculiarities, treating further related material can be found in [75, 74, 70, 42, 78, 16] and references therein.

2 Gibbs measures and quasilocality

In this section we will describe some definitions and facts we will need about the theory of Gibbs measures. For a more extensive treatment we refer to [21] or [74].

We will consider spin systems on a lattice Z^d, where in most cases we will take a single-spin space Ω_0 which is finite. The configuration space of the whole system is $\Omega = \Omega_0^{Z^d}$. Configurations will be denoted by σ or ω, and their coordinates at lattice site i are denoted by $\omega(i)$ or $\sigma(i)$. A (regular) interaction Φ is a collection of functions Φ_X on $\Omega_0^X, X \in Z^d$ which is translation invariant and satisfy:

$$\Sigma_{0 \in X} \ |\Phi_X|_\infty < \infty \tag{1}$$

Formally Hamiltonians are given by

$$H^\Phi = \Sigma_{X \in Z^d} \ \Phi_X \tag{2}$$

Under the above regularity condition these type of expressions make mathematical sense if the sum is taken over all subsets having non-empty intersections with a finite volume Λ. For regular interactions one can define Gibbs measures as probability measures on Ω having conditional probabilities which are described in terms of appropriate Boltzmann-Gibbs factors:

$$\frac{\mu(\sigma^1_\Lambda|\omega_{\Lambda^c})}{\mu(\sigma^2_\Lambda|\omega_{\Lambda^c})} = \exp -(\Sigma_X \ [\Phi_X(\sigma^1_\Lambda\omega_{\Lambda^c}) \ - \ \Phi_X(\sigma^2_\Lambda\omega_{\Lambda^c})]) \tag{3}$$

for each volume Λ, μ-almost every boundary condition ω_{Λ^c} outside Λ and each pair of configurations σ^1_Λ and σ^2_Λ in Λ. As long as Ω_0 is compact, there always exists at least one Gibbs measure for every regular interaction; the existence of more than one Gibbs measure is one definition of the occurrence of a first-order phase transition of some sort. Every Gibbs measure has the property that (for one of its versions) its conditional probabilities are continuous functions of the boundary condition ω_{Λ^c}. It is a non-trivial fact that this continuity, which goes by the name "quasilocality" or "almost Markovianness", in fact characterizes the Gibbs measures [67, 36], once one knows that all the conditional probabilities are bounded away from zero (that is, the measure is *nonnull* or has the *finite energy* property). In some examples it turns out to be possible to check this continuity (quasilocality) property quite explicitly. If a measure is a Gibbs measure for a regular interaction, this interaction is essentially uniquely determined.

A second characterization of Gibbs measures uses the variational principle expressing that in equilibrium a system minimizes its free energy. A probabilistic formulation of this fact naturally occurs in terms of the theory of large deviations. A (third level) large deviation rate function is up to a constant and a sign equal to a free energy density. To be precise, let μ be a translation invariant Gibbs measure, and let ν be an arbitrary translation invariant measure. Then the relative entropy density $i(\nu|\mu)$ can be defined as the limit:

$$i(\nu|\mu) = lim_{\Lambda\to Z^d} \ \frac{1}{|\Lambda|} \ I_\Lambda(\nu|\mu) \tag{4}$$

where

$$I_\Lambda(\nu|\mu) = \int log(\frac{d\nu_\Lambda}{d\mu_\Lambda})d\nu_\Lambda \tag{5}$$

and μ_Λ and ν_Λ are the restrictions of μ and ν to Ω_0^Λ. It has the property that $i(\nu|\mu) = 0$ if and only if the measure ν is a Gibbs measure for the same interaction as the base measure μ. We can use this result in applications if we know for example that a known measure ν cannot be a Gibbs measure for the same interaction as some measure μ we want to investigate. For example, if ν is a point measure, or if it is the case that ν is a product measure and μ is not, we can conclude from the statement: $i(\nu|\mu) = 0$, that μ lacks the Gibbs property.

For another method of proving that a measure is non-Gibbsian because of having the "wrong" type of (in this case too small) large deviation probabilities, see [64].

3 Renormalization-Group maps: some examples

We will mostly consider the standard nearest neighbor Ising model with (formal) Hamiltonian

$$H = \Sigma_{<i,j>} \ -\sigma(i)\sigma(j) \ - h\Sigma_i \ \sigma(i) \tag{6}$$

at inverse temperature β. The magnetic field strength is h. The dimension d in what follows will be at least 2.

We will consider various real-space Renormalization-Group or block-spin transformations which act on the Ising Gibbs measures. The question then is to find the renormalized interaction, that is the interaction associated to the transformed measure.

Although in applications the transformation needs to be iterated we will mostly restrict ourselves to considering a single transformation. Existence of the first step is of course necessary but far from sufficient for justification of an iterative procedure. As a further remark we mention that sometimes even if the first step is ill-defined, after repeated transformations a transformed interaction can be found[56].

We divide the lattice into a collection of non-overlapping blocks. A Renormalization-Group transformation defined at the level of measures will be a probability kernel

$$T(\omega'|\omega) = \Pi_{\text{blocks}} T(\omega'(j)|\omega(i);\ i\ \in \text{block}_j) \tag{7}$$

This means that the distribution of a block-spin depends only on the spins in the corresponding block, in other words the transformation is local in real space. The case of a deterministic transformation is included, by having a T which is either zero or one.

Renormalization-Group methods are widely in use to study phase transitions and in particular critical phenomena of various sorts (see for example [79, 48, 12, 19]). Some good more recent references in which the theory is explained, mostly at a physical level of rigour, but with more careful statements about what actually has and has not been proven are [22, 1].

1) One class of examples we will consider are (linear) block-average transformations. This means that the block-spins are the average spins in each block. Applied to Ising systems they suffer from the fact that the renormalized system has a different single-spin space from the original one. Despite this objection, the linearity makes these maps mathematically rather attractive, and they have often been considered. Because we are not iterating the transformation we need not worry too much about the single-spin space changing.

2) Majority rule and Kadanoff transformations.

In the case of majority rule transformations [59] applied to blocks containg an odd number of sites, the block spin is just given by the sign of the majority of the spins in the block. These transformations have been chosen often because of their numerical tractability.

The Kadanoff transformation is a soft version (a proper example of a stochastic transformation) of the majority rule:

$$T(\sigma'(j)|\sigma(i);\ i \in \text{block}_j)) = C\ \exp\,[p\sigma'(j)\Sigma_{i\in\text{block}_j}\ \sigma(i)] \tag{8}$$

In the limit in which p goes to ∞ the Kadanoff map reduces to a majority rule transformation.

3)Decimations and projections.

We will call a "decimation" taking the marginal of a measure restricted to the spins on a sublattice of the same dimension as the original system, (thus the block-spins are the spins in some periodic sub-lattice).

(A "projection" will mean taking the marginal to a lower-dimensional sublattice. Projections are not Renormalization-Group maps proper, but share some mathematical properties of Renormalizatio-Group maps. See [49, 50])

Although decimation transformation have the advantages both of being linear and of preserving the single-spin space, infinite iteration has the disadvantage that critical fixed points won't occur. However, this problem does not show up after a finite number of applications of these maps, so we will not need to worry about it.

4 The investigations of Griffiths and Pearce

Griffiths and Pearce [23, 24] seem to be the first investigators who looked seriously at the question whether renormalized Hamiltonians exist in a precise mathematical sense. They found that for some real-space transformations like decimation or Kadanoff transformations in the low-density regime (that means strong magnetic fields in Ising language) the Renormalization-Group map maps the Ising Gibbs measure on a Gibbs measure for an in principle computable interaction. They also found, both at phase transitions and near areas of the phase diagram where first order transitions occur, that there are regimes where the formal expression for renormalized interactions behaved in a peculiar way. These "peculiarities" were found for decimation, Kadanoff, and majority rule transformations. The problem underlying the peculiarities is the occurrence of phase transitions in the system, once it is constrained by prescribing some particular, rather atypical, block-spin configuration. This means that there can exist for these block-spin configurations long-range correlations in the presumed "short-wavelength degrees of freedom"- or "internal spins"-, which are to be integrated out in a Renormalization-Group map.

In their paper Griffiths and Pearce discuss various possible explanations of these "peculiarities":

P1)The renormalized interaction might not exist,

P2)it might exist but be a singular function of the original interaction,

P3)it might be non-quasilocal,
or

P4)the thermodynamic limit might be problematic.

Shortly after, the problem was studied by Israel [32]. He obtained (very)high temperature existence results, including approach to trivial fixed points, as well as an analysis of the decimation transformation at low temperature, indicating strongly that in that case the renormalized interaction does not exist. Israel's results convinced Griffiths that in fact possibility P1)— non-existence of the renormalized interaction— applies [25], however, it seems that most authors aware of their work interpreted the Griffiths-Pearce peculiarities as singular behaviour of the renormalized interactions (possibility P2)). See for example [29, 7, 31, 68]. Many authors did not even show awareness that the dtermination of renormalized interactions presented problems beyond mere computational difficulty. See the Appendix for some illustrations of this point.

Deep in the uniqueness regime the Renormalization-Group procedure appeared to be well-defined in some generality.

At that stage, various open questions formulated by Griffiths and Pearce and Israel were still left:

Q1) What is the nature of the "peculiarities"?

Q2) Can one say anything about the critical regime?

Q3) Do different transformations exhibit similar behaviour? In particular are momentum-space Renormalization-Group transformations similar to real-space transformations regarding the occurrence of "peculiarities"?

Q4) As the "peculiarities" are due to rather atypical spin-configurations, can one make the Renormalization-Group enterprise work, by considering only typical configurations, and thus work with appropriate approximations? Or, more generally, is there a framework in which one can one implement the whole Renormalization-Group machinery in a mathematically correct way?

5 Answered and unanswered questions

About question Q1 —the nature of the peculiarities— we have acquired some more insight. In [74] the Griffiths-Pearce study was taken up and further pursued. It was found, making use of the above-mentioned variational characterization of Gibbs measures, that the peculiarities could *not* be due to discontinuities in the Renormalization-Group maps, and that in the cases considered by Griffiths and Pearce the renormalized measures all have conditional probabilities with points of essential discontinuity. That is, they are *non-Gibbsian*. See also [35]. Thus a renormalized Hamiltonian *does not exist*. This despite many attempts to compute these —non-existent— renormalized Hamiltonians, and the various physically plausible and intuitively convincing conclusions, derived from such approximate computations.

In fact, those constrained block-spin configurations, pointed out by Griffiths and Pearce, for which the internal spins have phase transitions are precisely the points of discontinuity – non-quasilocality– for some conditional probability. The observation that the "peculiarities" were due to the violation of the quasilocality condition was in essence, though somewhat in a slightly implicit way, already made in Israel's analysis.

At first the non-Gibbsian examples were found at or near phase transitions, at sufficiently low temperatures. Then it was found [72] that decimation applied to many-state Potts models gives non-Gibbsian measures also above the transition temperature.

Somewhat surprisingly, it turned out that even deep in the uniqueness regime, the non-Gibbsianness can occur. This happens for example at low temperature for block-average [74], and majority rule [72] transformations in strong external fields, and it is even possible to devise transformations for which this happens at arbitrarily high temperatures [69]. On the other hand, it turns out that for Gibbs measures well in the uniqueness regime, a repeated application of a decimation transformation, even after composing with another Renormalization-Group map, leads again to a Gibbs measure, although applying the decimation only a few times may result in a non-Gibbsian measure [56, 57]. For related work in this direction see also [3]. Physically, this means that one cannot find a "reasonable" (more or less short-range, local) Hamiltonian.

About critical points (question Q2), Haller and Kennedy [28] obtained the first results, proving both for a decimation and a Kadanoff transformation example that a single Renormalization-Group map can map an area including a critical point to a set of renormalized interactions. There are strong indications for similar behaviour for other transformations [34, 2, 8, 60]. The indications are partly numerical,however, and fall short of a rigorous proof. See also the recent numerical work of [61]. On the other hand, counterexamples where a transformed critical measure is non-Gibbsian also exist [69, 70]

Another critical regime result is the observation of [13], that majority rule scaling limits of critical points above the upper critical dimension (when the critical behaviour is like that of a Gaussian) are non-Gibbsian.

Recently it was found that non-Gibbsian measures can also occur as a result of applying momentum-space transformations [71]. The conclusion of all the above is that different transformations can have very different behaviour.This is a sort of answer to question Q3, although in principle not a very informative one. In fact for applicability, if not for existence, something like this was already expected (compare for example Fisher's [18] remarks on "aptness" and focusability).

Regarding question Q4) —to find the right setting for implementing Renormalization-Group Theory— the issue is still essentially open. One approach which was stimulated by the late R.L. Dobrushin is related to the observation of Griffiths and Pearce that the block-spin configurations responsible for the peculiarities (the discontinuity points) are rather atypical. By removing them from configuration space, one might hope to be left with a viable theory. Such investigations have led to the notions of "almost" or "weak" Gibbs measures, the study of whose properties is being actively pursued [10, 11, 6, 49, 50, 77, 51, 17, 65, 40, 55, 44]. This approach is somewhat along the lines of Griffiths' and Pearce's possibility P3). See also the next section. Whether it is possible to describe Renormalization-Group flows in spaces of such interactions, while keeping a continuous connection between interactions describing a positively and a negatively magnetized state, is not at all clear.

As for projections, it is known that on the phase-transition line of the 2-dimensional Ising model the projection to Z of any Gibbs measure is non-Gibbsian [64]. In the whole uniqueness regime, except possibly at the critical point, this projection results in a Gibbsian measure [52, 42, 43]. In three dimensions the projected measures to two-dimensional planes are again non-Gibbsian on the transition line [17, 53, 50], however, now, due to the presumably existing surface (layering) transition between different Basuev states, one expects that the projected measures also in a small field will be non-Gibbsian [42, 43].

The composition of a projection and a decimation in the phase transition region gives rise to a new phenomenon, namely the possibility of a state-dependent result. The transformed plus and minus measures are Gibbs measures for different interactions, while the transformed mixed measures are non-Gibbsian [46].

6 Further results on non-Gibbsian measures

Further investigations in which non-Gibbsian measures were displayed, have been done about the random-cluster models of Fortuin and Kasteleyn [62, 26, 4, 27, 65], about the Fuzzy Potts image analysis model [54], and about various non-equilibrium models [66, 78, 58, 57].

The non-Gibbsian character of the various measures considered comes often as an unwelcome surprise. A description in terms of effective, coarse-grained or renormalized potentials is often convenient, and even seems essential for some applications. Thus, the fact that such a description is not available thus can be a severe drawback. As remarked earlier there have been attempts to tame the non-Gibbsian pathologies, and here we want to give a short comparison of how far one gets with some of those attempts.

1) The fact that the constraints which act as points of discontinuity often involve configurations which are very untypical for the measure under consideration, suggested a notion of *almost* Gibbsian or *weakly* Gibbsian measures. These are measures whose conditional probabilities are either continuous only on a set of full measure or can be written in terms of an interaction which is summable only on a set of full measure. Intuitively, the difference is that in one case the "good" configurations can shield off *all* influences from infinitely far away, and in the other case only *almost all* influences. The weakly Gibbsian approach was first suggested by Dobrushin to various people; his own version was published only later[9, 10, 11]. An early definition of almost Gibbsianness appeared in print in [47],see also [17], [54, 55, 49] and [62] for further developments. Some examples of measures which are at the worst almost Gibbsian measures in this sense are decimated or projected Gibbs measures in an external field, random-cluster measures on regular lattices, and low temperature fuzzy Potts measures. In the random-cluster measures one can actually identify explicitly all bond configurations which give rise to the non-quasilocality. They are precisely those configurations in which (possibly after a local change) more than one infinite cluster coexist.

On a tree, because of the possible coexistence of infinitely many infinite clusters with positive probability, the random-cluster measure can violate the weak non-Gibbsianness condition and be strongly non-Gibbsian [27].

Dobrushin [9, 55, 10, 11] showed that for a projected pure phase on the coexistence line of the 2-dimensional Ising model it is possible to find an almost everywhere defined interaction, hence these measures are weakly Gibbsian. His approach, which is via low-temperature expansions, provides a way of obtaining good control for the non-Gibbsian projection. For similar ideas in a Renormalization-Group setting see [6, 40, 45].

For some Renormalization-Group examples an investigation via low temperature expansions into the possibility of recognizing non-Gibbsianness was started in [63].

Another simple counter-example of a strongly non-quasilocal measure, where each configuration can act as a point of discontinuity, is a mixture of two Gibbs measures for different interactions [76].

2) Stability under decimation (and other transformations).

In [56, 57] it was shown how decimating once renormalized non-Gibbsian measures results in Gibbs measures again after a sufficiently large number of iterations. These often decimated measures are in the high-temperature regime, in which the usually applied Renormalization-Group maps are well-defined (this does not hold true for all maps though, in view of the highly non-linear example given before).

On the other hand, in examples where the non-Gibbsian property is associated with large deviation properties which are not compatible with a Gibbsian character (this holds for example for projected Gaussians, invariant measures of the voter and the Martinelli-Scoppola model[38, 39, 58]) the non-Gibbsian property survives all sort of transformations [57, 76]. The argument is that when some obviously non-Gibbsian measure has a rate function zero with respect to the measure under consideration, this property is generally preserved ([74] Section 3.2 and 3.3).

The family of projected Gaussians include scaling limits for majority-rule transformations in high dimensions [13]. The transformation of those scaling limits is,heuristically, interpreted as making a move from a fixed point in what is usually called a "redundant" direction (cf. Wegner's contribution to [12]) in some space of Hamiltonians. Here, of course, the whole point is that such Hamiltonians do not exist.

3) The two criteria mentioned above are distinct. A simple one-dimensial example due to J. van den Berg [45] gives a one-dependent measure which has a set of discontinuity points of full measure, but due to the one-dependence the measure becomes after decimation independent and therefore trivially Gibbsian. In the opposite direction, there exist examples of measures whose non-Gibbsianness is robust, although they are weakly, and even almost Gibbsian[77].

7 Conclusions

We have by now managed well to understand the Griffiths-Pearce peculiarities in the sense that we can identify mathematically their nature. However, how to get around them, and make Renormalization-Group theory mathematically respectable, is still a task requiring a lot of further work. Renormalization-Group ideas have of course often been inspirational, also for various rigorous analyses. Renormalization-Group implementations on spin models, which both for numerical and pedagogical convenience have often been treated by Renormalization-Group methods, run into difficulties which seem hard to avoid. An implementation of Renormalization-Group ideas on contour models looks more promising, at least for the description of first-order phase transitions [20, 5].

In non-equilibrium statistical mechanics there are still many open questions about the occurrence of non-Gibbsian measures. The term non-Gibbsian or non-reversible is often used for invariant measures in systems in which there is no detailed balance [41, 15, 14]. It is an open question to what extent such measures are non-Gibbsian in the sense we have described here. It has been conjectured that such measures for which there is no detailed balance are quite generally non-Gibbsian in systems with a stochastic dynamics, see for example [39] or [15], Appendix 1; on the other hand it has been predicted that non-Gibbsian measures are rather exceptional ([41], Open problem IV.7.5, p.224) at least for non-reversible spin-flip processes under the assumptions of rates which are bounded away from zero. The examples we have are for the moment too few to develop a good intuition on this point.

Non-Gibbsian measures occur in quite different areas of statistical mechanics, and can have quite different properties. By now it seems somewhat surprising that it took so long to appreciatee the fact that the Gibbs property is rather special, in particular in view of Israel's [33] result that in the set of all translation invariant (ergodic, nonnull) measures, Gibbs measures are exceptional.

8 Appendix: Some literature

As an illustration that the heuristic character of Renormalization-Group theory, and in particular the need to consider the existence problem of (approximately) local renormalized interactions was

recognized by various practitioners I mention the following quotes:
One cannot write a renormalization cook book (K.G. Wilson, cited by Niemeijer and van Leeuwen [12])
The notion of renormalization group is not well-defined ([1], opening sentence).
It is dangerous to proceed without thinking about the physics [22]
...the locality [of the renormalized interactions] is a non-trivial problem which will not be discussed further [79].
A Renormalization Group for a space of Hamiltonians should satisfy the following [18]:
A) Existence in the thermodynamic limit,...
C) Spatial locality...
In the words of Lebowitz: On the cautionary side one should remember that there are still some serious open problems concerning the nature of the RG transformation of Hamiltonians for statistical mechanical systems, i.e. for critical phenomena. A lot of mathematical work remains to be done to make it into a well-defined theory of phases transitions [37].

As an illustration that on the other hand the occurrence of the above-mentioned problems has not been generally recognized, leading to some incorrect or at least misleading statements, I quote:
The renormalization-group operator ... transforms an arbitrary system in the [interaction] space ... into another system in the space .... [68]
[the set of coupling constants gives rise]..the most general form of the Hamiltonian... [30]
Further iterations of the renormalization group will generate long-range and multi-spin interactions of arbitrary complexity [80].
...the space of Ising Hamiltonians in zero field may be specified by the set of all possible spin-coupling constants, and is *closed* [Fisher's emphasis] under the decimation transformation [19].

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