# REDUCIBLE CONNECTIONS IN GAUGE THEORIES

Giuseppe Gaeta*
*Dipartimento di Fisica, Università di Roma I*
*I–00185 Roma (Italy)*
giuseppe.gaeta@roma1.infn.it

Paola Morando
*Dipartimento di Matematica, Politecnico di Torino*
*I–10139 Torino (Italy)*
morando@polito.it

**Abstract.** When studying gauge theories (e.g. with finite energy conditions), attention is traditionally reserved to the subset of irreducible connections, which is open and dense in the full space of connections. We point out that in general the residual set of reducible connections contains critical points of the gauge functionals, which moreover are in general the only ones common to all theories with a given symmetry, i.e. those determined by the symmetry and the geometry of the problem alone, and not by the specific choice of the functional.

* Present address: Department of Mathematics, Loughborough University, Loughborough LE11 3TU (England)

Last modified 29/11/97
P.A.C.S. numbers: 11.15.Ex ; 02.20.Tw , 02.30.Wd , 02.40.Vh

## 1. Introduction.

When we study a pure gauge theory (e.g., but not necessarily, of Yang-Mills type), the central object is a (gauge-invariant) functional $L$ defined on a space of connections $\mathcal{A}$ of the associated bundle $\mathtt{ad}P$ to the principal $G$-bundle $P$ over the base space $B$ (usually the four-dimensional spacetime or a compactification of it) which defines the gauge invariance of the theory.

This space of connections $\mathcal{A}$ is not the full set of connections, but a subset of it; this is typically a Sobolev space (which ensures a number of technically relevant properties) chosen in agreement with physical requirements, usually a finite energy condition appropriate for $L$ and the physical nature of the problem.

However, it is by now customary to consider not the full Sobolev space $\mathcal{A}$, but a subset of it, which is open and dense in $\mathcal{A}$ and which is technically more manageable to study. This subset $\mathcal{A}^*$ corresponds to the so-called *irreducible connections* (relevant definitions will be recalled below).

Reducing one's attention to $\mathcal{A}^*$ rather than to the full $\mathcal{A}$ is justified on the one side by the fact that this corresponds to generic connections – we have already mentioned that $\mathcal{A}^*$ is open and dense in $\mathcal{A}$ – and on the other by the fact that in this way the analysis is greatly simplified, as the connections in $\mathcal{A}_0 \equiv \mathcal{A}\backslash\mathcal{A}^*$ are indeed singular in many senses, and one only overlooks a set of measure zero. Moreover, the topology of $\mathcal{A}^*$ conveys information on the structure of the "erased part" $\mathcal{A}_0$ as well.

However, to the eyes of a reader familiar with the theory of symmetry breaking in finite dimensions – e.g. in theories defined by minimization of a finite dimensional $G$-invariant potential $V(x)$, with $G$ a compact Lie group and $x \in M \subseteq R^n$ – this reduction could cause some perplexity: in this setting, it is quite well known that there is a "generic stratum" $M^*$, open and dense in $M$ (or similarly when considering the set $\Omega \simeq M/G$ of $G$-orbits), but it is also known that generically the critical point of $V$ lie precisely in $M\backslash M^*$, and more precisely in the "most singular" (in a precise sense to be defined below) strata.

The simplest example of this situation is trivial: it suffices to consider potentials $V$ defined on $R$ and which are invariant under $G = Z_2$ acting as $g(x) = -x$: the set $M^*$ is given by $R\backslash\{0\}$, but the origin is a critical point for all such potentials, and indeed the only generic critical point for this class of $Z_2$-invariant potentials.

As a less trivial application of the general finite-dimensional theory [1], one can consider the well known – and celebrated – works of Michel and Radicati on the breaking of $SU(3)$ symmetry in hadronic interactions [2], in which the physical particles are shown to be in correspondence precisely with points in the most singular strata, and thus in particular with orbits which lie in $M\backslash M^*$.

The Michel's geometric theory of symmetry breaking has been recently extended [3] to the full setting of gauge theories (both pure ones and with matter fields, and not necessarily of

Yang-Mills type and/or defined by first order functionals), leading to a complete analogy with the original results of Michel in the finite dimensional setting.

This extensions shows in particular that gauge orbits which are isolated in their stratum (again, relevant definitions will be introduced below) are necessarily critical points of any gauge invariant functional. But such gauge orbits are necessarily *not* in the generic set $\mathcal{A}^*$ – they are actually the "most singular" among the singular orbits – and thus by restricting to $\mathcal{A}^*$ one is overlooking a whole class of critical points, which moreover are the only critical points which are common to all the gauge theories with the same gauge invariance, i.e. which are fixed by the symmetry (and geometry) of the problem itself.

The purpose of this note is to illustrate the above statement, avoiding to go into excessive mathematical detail (a mathematically complete, detailed and rigorous proof is published elsewhere [3]) and focusing instead on the physical side of the question. In particular, we would like to attract the attention of the reader on the fact that – except for quite special groups such as $SU(2)$ – one cannot limit his/her attention to $\mathcal{A}^*$, as in doing this one is necessarily forgetting critical points of the theory.

It should be stressed that our analysis is only at the classical level; when one considers the quantum theory – which we do *not* do, neither here nor elsewhere – one has to integrate over the full space of connections, and by considering only $M^*$ one is indeed leaving out of the integration only a set of measure zero (corresponding to singular orbits).

However, if one is approaching the quantum theory by a WKB or related approximations, it becomes specially relevant even in this case to know the critical points – and, we stress, not only the stable ones – of the classical theory, which are used to start the perturbative expansion, so that again it is quite dangerous to sistematically leave out of consideration the set $\mathcal{A}\backslash\mathcal{A}^*$, which is now known to contain generic critical points.

## 2. Irreducible connections.

We consider a finite dimensional connected riemannian manifold $B$ (e.g. $R^4$ or $S^4$) and a compact semisimple Lie group $G$ [e.g. $SU(n)$] with Lie algebra $\mathcal{G}$; and a principal bundle $P$ over $B$, with projection $\pi : P \to B$ and fiber $\pi^{-1}(x) = G$. We also consider the corresponding adjoint bundle $\mathtt{ad}P$, with fiber $\mathcal{G}$ and $G$-action on it defined by the adjoint action of $G$ on its Lie algebra $\mathcal{G}$.

In the affine space $\mathcal{C}$ of connections on $P$ we can select a reference point (i.e. connection) $A^0$; with this we can work in the vector space of connection forms, i.e. in the space $\mathcal{A}$ of one-forms on $B$ with values in $\mathcal{G}$: to a one-form $\alpha = A_\mu \mathrm{d}x^\mu$ corresponds the connection with components $A_\mu$, and the covariant derivative $\nabla_\mu = \partial_\mu + A_\mu$. We will, for ease of notation, speak improperly of "the connection $\alpha$".

We consider the associated bundle $\mathtt{ad}P$ on $B$, and the space of differentiable sections $\Gamma$ on it; this is the gauge group modelled on $G$. An element $\gamma \in \Gamma$ acts naturally on connections by conjugation: if $\gamma$ is, in local coordinates, given by $g(x)$, then $\gamma : \alpha \to \gamma(\alpha) = \beta$ given in local coordinates by $\beta = B_\mu \mathrm{d}x^\mu$ with

$$B_\mu(x) = g(x) \cdot A_\mu(x) \cdot g^{-1}(x) - (\partial_\mu g)(x) \cdot g^{-1}(x) \ . \tag{1}$$

Thus the gauge isotropy subgroup $\Gamma_\alpha$ of the connection $\alpha$, which is obviously defined as

$$\Gamma_\alpha = \{\gamma \in \Gamma \ : \ \gamma(\alpha) = \alpha\} \ , \tag{2}$$

is given in local coordinates by the $g(x)$ such that $(\partial_\mu g)(x) = [g(x), A_\mu(x)]$, as follows by requiring that $B_\mu = A_\mu$ in (1).

As mentioned in the introduction, one prefers – for reasons of both mathematical and physical nature, the latter ones being usually finite energy conditions – to consider Sobolev classes of connections and gauge sections; by a suitable choice of the Sobolev norms and of the Sobolev class to consider (depending in particular on the dimension $k$ of the base space $B$), one can work properly in this setting; in this note we will not discuss this analytical aspect of the question, referring to [3] for detail, and focus on the main geometrical issue: here we only want to reassure the reader that the analytical side is not forgotten, and that $\mathcal{A}$ and $\Gamma$ should more precisely be thought of as subsets of the full sets of connections and gauge sections, which are Sobolev spaces of class, respectively, $k$ and $k-1$ in suitable norm [4,5]

It is a classical and well known result [6] that $\Gamma_\alpha$ is easily characterized in terms of any reference point $p_0 \in P$, with $\pi(p_0) = x_0$, as the set of sections in $\mathcal{G}$ which take in $x_0$ values in $C_G[H_\alpha(p_0)]$, the centralizer in $G$ of the holonomy group $H_\alpha(p_0)$ of the connection $\alpha$ at $p_0$ – that is, such that $g(x_0) \in C_G[H_\alpha(p_0)] \subseteq G$ – and which are covariantly constant along the connection $\alpha$ itself.

This means that for any $\alpha$ the gauge isotropy subgroup $\Gamma_\alpha$ is isomorphic to a compact group, but it should be noticed that the isomorphism depends on $\alpha$ itself.

A dichotomy immediately arises in $\mathcal{A}$: the connections for which $\Gamma_\alpha$ is nontrivial – i.e. $C_G[H_\alpha(p_0)]$ does not reduce to $\{e\}$ for all $p_0 \in P$ – are called *reducible* (we denote their set by $\mathcal{A}_0 \subset \mathcal{A}$), and one can obtain a reduction of $P$ under them (this means that $P$ can be decomposed into the disjoint union of subbundles, invariant under the connection); the others are called *irreducible*, and we denote their set by $\mathcal{A}^* \subset \mathcal{A}$.

It is a general result that, under the above assumptions on $B$, $G$, $P$ and $\Gamma$, the set $\mathcal{A}^*$ is open and dense in $\mathcal{A}$ (e.g., for $G = SU(2)$ any nonzero connection is in $\mathcal{A}^*$ [4]); we also say that $\mathcal{A}^*$ constitutes the generic *stratum* in $\mathcal{A}$.

The set $\mathcal{A}^*$ is obviously easier to study than the full set $\mathcal{A}$; actually, one can prove [6] that it is a smooth Hilbert manifold, and a principal $G$-bundle [7]. Moreover, as it is open and dense in $\mathcal{A}$, it seems quite reasonable to confine attention to its study; indeed, this is what is done in a vaste body of literature on gauge theories.

## 3. Reducible connections.

We want now to briefly present a recent result providing an identification of points which are critical for any gauge functional (once $P$ – and thus $B$ and $G$ – is given); these turn out to correspond to reducible connections.

We will not give proofs, referring again to [3], but we need however some work to introduce the notation suitable to formulate the result.

We consider functionals $L : \mathcal{A} \to R$ which are invariant under $\Gamma$, i.e. such that $L[\alpha] = L[\gamma(\alpha)]$ for all $\gamma \in \Gamma$ and all $\alpha \in \mathcal{A}$; we assume these functionals are defined by a local density $\mathcal{L}[A;x]$, function of $A(x)$ and its derivatives (up to any finite order) at the point $x \in B$, and maybe explicitely dependent on $x$ itself, so that

$$L[A] \ = \ \int_B \mathcal{L}[A;x] \, \mathrm{d}^k x \ . \tag{3}$$

The $\Gamma$-invariance of $L$ guarantees that critical points of $L$ come in $\Gamma$-orbits. We say that a $\Gamma$-orbit $\Omega$ is $\Gamma$-*critical* if it is an orbit of critical points for any $\Gamma$-invariant functional $L$.

To any $\alpha \in \mathcal{A}$ we associate the isotropy subgroup $\Gamma_\alpha$ defined above; through this, $\mathcal{A}$ can be partitioned into *isotropy types*, or *strata*, defined as sets of connections having isotropy subgroups belonging to the same conjugacy class under conjugation in $\Gamma$: thus the stratum $\Sigma(\alpha)$ of $\alpha$ is defined as

$$\Sigma(\alpha) \ = \ \{ \beta \in \mathcal{A} \ : \ \Gamma_\beta = \gamma \, \Gamma_\alpha \, \gamma^{-1} \ , \ \gamma \in \Gamma \} \ . \tag{4}$$

Obviously, $\Gamma(\alpha) \subseteq \Sigma(\alpha)$; notice that in the present gauge setting the set of strata is countable. The strata are smooth manifolds, and principal bundles (see [7] for detail and some subtleties involved in the bundle structure; further detail on the geometry of gauge orbits is given in [8]).

The topology defined in $\mathcal{A}$ induces a topology in the strata (as $G$ is compact and thus acts regularly, in particular on its Lie algebra); thus it makes sense to speak of an orbit $\Omega$ which is *isolated in its stratum*; this intuitive concept can be defined more precisely using slices and tubular neighbourhoods around $\Gamma$-orbits [3], and amounts to saying that no other orbit belonging to the same stratum is near (in the natural topology) to $\Omega$.

We have then the following result, proved in full detail in [3], which is an immediate extension – including the method of proof – of a classical result obtained by Louis Michel [1] in 1971, motivated by the Physics of hadronic interactions [2]:

**Theorem.** *A $\Gamma$ gauge orbit $\Omega \subset \mathcal{A}$ is $\Gamma$-critical if and only if it is isolated in its stratum.*

As the stratum of irreducible connections is open and dense, it is obvious that $\Gamma$-critical orbits, if they exist, do necessarily correspond to reducible connections.

It should be mentioned that, as also discussed in detail in [3], the same result extends to theories with matter fields (and actually also to the case, maybe less relevant physically, of gauge-equivariant equations which do not come from a variational principle [9]). Similarly, the theorem given above can be combined with bifurcation theory to describe symmetry breaking arising in second-order phase transitions [3].

## 4. Relevance for Physics.

We have thus, after introducing the (minimal) needed notation, stated our result, and noticed that it points out to critical connections – and in particular $\Gamma$-critical $\Gamma$-orbits of connections – which are reducible, i.e. which would be overlooked if restricting attention to $\mathcal{A}^*$ rather than considering the full $\mathcal{A}$. Actually, we are in the opposite situation: these are necessarily in $\mathcal{A}_0 = \mathcal{A} \backslash \mathcal{A}^*$, so that our approach does naturally point in a direction which is "orthogonal" to the standard one.

In the present section we want to argue that such a result is not a mathematical curiosity, or even a mathematically relevant fact, but it is also relevant from the point of view of Physics.

First of all, we stress that, as already mentioned, there are gauge orbits which are critical for *any* smooth functional with the required gauge invariance: thus these critical points are directly related to the symmetry rather than to the detailed choice of the functional. In this sense, i.e. from a symmetry point of view, these $\Gamma$-critical orbits (which correspond necessarily to reducible connections, see above) are more fundamental than any other solution; in our opinion, this would already be by itself a sufficient reason to study them in detail.

In this respect, it should be recalled what is the situation in the finite-dimensional approximation for strong interactions: here the theory is described by a $G$-invariant potential on the Lie algebra $\mathcal{G}$; we have $G = SU(3)$ acting on its algebra by the adjoint representation, and we should fix the normalization condition $|x| = 1$ (for $x \in R^8 \approx \mathcal{G}$). When we look at the $G$-orbit space, we deal with a subset of $R^2$ (corresponding to a Cartan subalgebra of $\mathcal{G}$; the $G$-invariants can be chosen e.g. as the two independent eigenvalues of the $su(3)$ matrices), and again by normalization we restrict to the unit circle. On this there are points which correspond to orbits isolated in their stratum, i.e. to $G$-critical orbits: these, together with the one corresponding to the origin, give precisely the $SU(3)$ octet of hadronic interactions [2,10].

It should be stressed that the same situation is encountered if we abandon the finite-dimensional (i.e. potential) approximation, and consider the gauge theory in its full (functional) setting: in this case one obtain, with an analysis which is only slightly more complicated than the finite dimensional one – and resorting also to the "Symmetric Criticality Principle" of Palais [11] – again that the $\Gamma$-critical orbits corresponds to covariantly constant gauge fields representing the physical particles of the $SU(3)$ octet [3].

In a sense, it is surprising that the many works adopting "automatically" the restriction to irreducible connections avoid to mention the fact that these – relevant and very simple – solutions lie outside of such a domain. On the other side, the technical complication met in attempting a detailed analysis of the space of reducible connections [7,12] can explain why this is so often avoided.

In a way, the approach proposed here suggest to start from the most singular strata rather than approaching them from the generic one; this is so much more reasonable as the theorem reported above focuses our attention not just on reducible connections, but precisely on the "most reducible" ones. These are the ones which are more difficult to study in the standard way, i.e. as the border of the generic stratum – and of any other one – but being at the other extremum of the range they can be profitably tackled in their own sake.

A strength and at the same time a weakness of the approach followed here is that our result holds for *any* gauge invariant functional, i.e. not only for gauge functionals of Yang-Mills type. This is a strength because of the greater generality, but a weakness in that we fail to make use of the specificity of the YM, physically relevant, case.

Similarly, one is not restricted to functionals of first order (i.e. depending on the fields and derivatives of order up to one alone), but could consider functionals of any finite order: again, this is a more general setting, but – precisely for this – it does not take full advantage of the specific YM structure.

It should also be mentioned that in this way we have no limitation to minima and/or selfdual or anti-selfdual critical points, but can obtain any kind of critical point.

In this respect, it should maybe be recalled that critical points which are not minima can be quite relevant physically, e.g. in the WKB or related approximations when dealing with the quantum case. Also, the theorems of Bourguignon and Lawson [13] stating that all the weakly stable critical points of YM functionals are selfdual or anti-selfdual, apply under precise conditions, which on the one side include the physically relevant case but on the other are not generic: thus, they only hold in dimension four and for a restricted class of gauge groups [including the physically relevant ones, $SU(2)$ and $SU(3)$]; and do not deal with unstable critical points.

In principle, the present approach could also be useful in determining the metric in the moduli space, as it localizes the critical points in strata and thus could help in determining a parametrization (maybe using considerations on the Hilbert basis of invariants for $G$).

Finally, it should be mentioned that the approach proposed here, as the whole Michel theory of which this is just a small extension, relies simply on the combination of group-invariant geometry and of topology, provided there is a suitable "agreement" between the symmetry and the topology, i.e. essentially that the projection to (gauge) orbit space "agrees" with the topology of the space in which the theory is defined. This is a quite general situation, and thus it is not surprising that the range of applications of Michel theory is correspondingly ample. This same consideration also suggests that Michel theory could maybe be formulated in a more "intrinsic" way, i.e. (roughly speaking) in terms of Atyiah-Bott rather than Bredon equivariant cohomology (this remark is due to L.A. Ibort).

It is obvious, but maybe equally worth mentioning, that all our discussion is at a purely classical level, i.e. we have only looked at critical points of the classical gauge functionals, and not at the corresponding functional integral; obviously, as already recalled, the classical critical points can be used to start a perturbative expansion (WKB method), which in some cases (as in the Duistermaat-Heckmann theory) can even become exact.

## 5. Conclusions.

In the early days of gauge theories it was noticed that the subspace $\mathcal{A}^*$ of irreducible connections is on the one side much easier to study than the full space $\mathcal{A}$ of connections (in the chosen Sobolev class), and on the other side that $\mathcal{A}^*$ is open and dense in $\mathcal{A}$. Also, the topology of $\mathcal{A}^*$ conveys information on the "erased part" $\mathcal{A}\backslash\mathcal{A}^*$, i.e. on the set of reducible connections. Thus, it was suggested that many relevant informations – as many as possible – should be extracted directly from $\mathcal{A}^*$, without embarking in tackling directly the complex geometry of $\mathcal{A}/\Gamma$.

We have pointed out here that many relevant features, in particular $\Gamma$-critical orbits, not only lie in the set of reducible connections, but actually in the most singular strata. This suggest that, rather than starting from the study of the generic stratum (irreducible connections), one could instead study directly the most singular strata, and maybe the less and less singular strata in order to analyze the successive symmetry breakings.

This is, of course, in agreement with the philosophy underlining Michel's symmetry breaking theory, according to which – in Louis Michel's words – "quite often Physics is in the most singular strata".

**Acknowledgements.**

The work described here was performed at – and benefited from support by – the M.F.O. in Oberwolfach (under the *Research in Pairs* program, supported by Volkswagen Stiftung) and the I.H.E.S. in Bures sur Yvette: we warmly thank these Institutions and their Directors. We also acknowledge financial support from CNR-GNFM (Italy).

We would like to thank some friends for useful discussions, and in particular C. Bachas and A. Chakrabarti in the key stage of this work, and C. Hayat-Legrand and L.A. Ibort when (we wrongly thought) it was completed. The encouragement of J.P. Bourguignon and of L. Michel was essential at several stages.

**References.**


[1] L. Michel, "Points critiques de fonctions invariantes sur une G-variété", *Comptes Rendus Acad. Sci. Paris* **272-A** (1971), 433-436.
L. Michel, "Nonlinear group action. Smooth action of compact Lie groups on manifolds", in *Statistical Mechanics and Field Theory*, R.N. Sen and C. Weil eds., Israel University Press, Jerusalem 1971.
L. Michel, "Les brisure spontanées de symétrie en physique", *J. Phys. (Paris)* **36** (1975), C7-41.
L. Michel, "Symmetry defects and broken symmetry. Configurations. Hidden symmetry", *Rev. Mod. Phys.* **52** (1980), 617-651.

[2] L. Michel and L. Radicati, "Properties of the breaking of hadronic internal symmetry", *Ann. Phys. (N.Y.)* **66** (1971), 758-783.
L. Michel and L. Radicati, "The geometry of the octet", *Ann. I.H.P.* **18** (1973), 185.

[3] G. Gaeta and P. Morando, "Michel theory of symmetry breaking and gauge theories", preprint `IHES/P/97/8`; *Ann. Phys. (N.Y.)* **260**, 149-170.

[4] M.S. Narashiman and T.R. Ramadas, "Geometry of SU(2) gauge fields", *Comm. Math. Phys.* **67** (1979), 121-136.

[5] G. Dell'Antonio and D. Zwanziger, "Every gauge orbit passes inside the Gribov horizon", *Comm. Math. Phys.* **138** (1981), 291-299.

[6] I.M. Singer, "Some remarks on the Gribov ambiguity", *Comm. Math. Phys.* **60** (1978), 7-12.
I.M. Singer, "The geometry of the orbit space for non-abelian gauge theories" *Physica Scripta* **24** (1981), 817-820.

[7] A. Heil, A. Kersch, N. Papadopolous, B. Reifenhauser and F. Scheck, "Structure of the space of reducible connexions for Yang-Mills theories", *J. Geom. Phys.* **7** (1990), 489-505.

[8] H.B. Lawson , *The theory of gauge fields in four dimensions*, A.M.S. (Providence) 1985.
P. Cotta-Ramusino and C. Reina, "The action of the group of bundle automorphisms on the space of connections and the geometry of gauge theories", *J. Geom. Phys.* **1** (1984), 121-155.
W. Kondracki and P. Sadowski, "Geometric structure on the orbit space of gauge connections", *J. Geom. Phys.* **3** (1986), 421-434.
W. Kondracki and J.S. Rogulski, "On the stratification of orbit space for the action of automorphisms on connections", *Diss. Math.* **250** (1986), 1-62.
K.B. Marathe and G. Martucci, "The geometry of gauge fields", *J. Geom. Phys.* **6** (1989), 1-106.
M.C. Abbati, R. Cirelli, A. Manià and P. Michor, "The Lie group of automorphisms of a principal bundle", *J. Geom. Phys.* **6** (1989), 215-235.

M.C. Abbati, R. Cirelli and A. Manià, "The orbit space of the action of gauge transformation group on connections", *J. Geom. Phys.* **6** (1989), 537-558.

[9] G. Gaeta and P. Morando, "Commuting-flow symmetries and common solutions to differential equations with common symmetries"; preprint `IHES/P/97/9`, to appear in *J. Phys. A.*

[10] G. Gaeta, "Bifurcation and symmetry breaking", *Phys. Rep.* **189** (1990), n.1., 1-87.

[11] R.S. Palais, "The principle of symmetric criticality", *Comm. Math. Phys.* **69** (1979), 19-30.
R.S. Palais, "Applications of the symmetric criticality principle in mathematical physics and differential geometry", in *Proceedings of the 1981 Shanghai symposium on differential geometry and differential equations*, Gu Chaohao ed., Science Press, Beijing, 1984.

[12] C.J. Isham, "Space-time topology and spontaneous symmetry breaking", *J. Phys. A.* **14** (1981), 2943-2956.

[13] J.P. Bourguignon and H.B. Lawson, "Stability and isolation phenomena for Yang-Mills fields", *Comm. Math. Phys.* **79** (1981), 189-230.
J.P. Bourguignon, H.B. Lawson and J. Simons, "Stability and gap phenomena for Yang-Mills fields", *Proc. Natl. Acad. Sci. USA* **76** (1979), 1550-1553.