HIGH-ENERGY NUCLEAR PHYSICS WITH LORENTZ SYMMETRY VIOLATION

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Abstract

If textbook Lorentz invariance is actually a property of the equations describing a sector of the excitations of vacuum above some critical distance scale, several sectors of matter with different critical speeds in vacuum can coexist and an absolute rest frame (the vacuum rest frame) may exist without contradicting the apparent Lorentz invariance felt by "ordinary" particles (particles with critical speed in vacuum equal to c , the speed of light). Sectorial Lorentz invariance, reflected by the fact that all particles of a given dynamical sector have the same critical speed in vacuum, will then be an expression of a fundamental sectorial symmetry (e.g. preonic grand unification or extended supersymmetry) protecting a parameter of the equations of motion. Furthermore, the sectorial Lorentz symmetry may be only a low-energy limit, in the same way as the relation ω (frequency) = c_s (speed of sound) k (wave vector) holds for low-energy phonons in a crystal. In this context, phenomena such as the absence of Greisen-Zatsepin-Kuzmin cutoff for protons and nuclei and the stability of unstable particles (e.g. neutron, several nuclei...) at very high energy are basic properties of a wide class of noncausal models where local Lorentz invariance is broken introducing a fundamental length. Observable phenomena are expected at very short wavelength scales, even if Lorentz symmetry violation remains invisible to standard low-energy tests. We present a detailed discussion of the implications of Lorentz symmetry violation for very high-energy nuclear physics.

1 Introduction

In previous papers (Gonzalez-Mestres, 1997a and 1997b) we suggested that, as a consequence of nonlocal dynamics at Planck scale or at some other fundamental length scale, Lorentz symmetry violation can result in a modification of the equation relating energy and momentum which would write in the vacuum rest frame:

$$E \quad = \quad (2\pi)^{-1} \ h \ c \ a^{-1} \ e \ (k \ a) \tag{1}$$

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where E is the energy of the particle, h the Planck constant, c the speed of light, a a fundamental length scale (that we can naturally identify with the Planck length, but other choices of the fundamental distance scale are possible), k the wave vector modulus and $[e\ (k\ a)]^2$ is a convex function of $(k\ a)^2$ obtained from nonlocal vacuum dynamics.

Rather generally, we find that, at wave vector scales below the inverse of the fundamental length scale, Lorentz symmetry violation in relativistic kinematics can be parameterized writing:

$$e\ (k\ a) \ \simeq \ [(k\ a)^2 \ - \ \alpha\ (k\ a)^4 \ + \ (2\pi\ a)^2\ h^{-2}\ m^2\ c^2]^{1/2} \tag{2}$$

where α is a positive constant between 10^{-1} and 10^{-2} if the nonlocal effect occurs at leading level at the fundamental length scale (which we hereafter assume unless the contrary is explicitly stated). At high energy, we can write:

$$e\ (k\ a) \ \simeq \ k\ a\ [1 \ - \ \alpha\ (k\ a)^2/2] \ + \ 2\ \pi^2\ h^{-2}\ k^{-1}\ a\ m^2\ c^2 \tag{3}$$

and, in any case, we expect observable kinematical effects when the deformation term $\alpha(ka)^3/2$ becomes as large as the mass term $2\ \pi^2\ h^{-2}\ k^{-1}\ a\ m^2\ c^2$. Assuming that, apart form the value of the mass, expression (2) is universal for all existing particles whose critical speed in vacuum is equal to the speed of light in the Lorentz-invariant limit (i.e. assuming universal values of c and α) , we found several important effects (Gonzalez-Mestres, 1997c) that can turn the study of very high-energy cosmic rays into a unique frame to test special relativity and physics at Planck scale:

a) The Greisen-Zatsepin-Kuzmin (GZK) cutoff on very high-energy cosmic protons and nuclei (Greisen, 1966; Zatsepin and Kuzmin, 1966) does no longer apply (Gonzalez-Mestres, 1997a and 1997b). Very high-energy cosmic rays originating from most of the presently observable Universe can reach the earth and generate the highest-energy detected events. Indeed, fits to data below $E\ =\ 10^{20}\ eV$ using standard relativistic kinematics (e.g. Dova, Epele and Hojvat, 1997) predict a sharp fall of the event rate at this energy, in contradiction with data (Bird et al., 1993 and 1996; Hayashida et al., 1994 and 1997; Yoshida et al., 1995) which suggest that events above $10^{20}\ eV$ are produced at a significant rate. Lorentz symmetry violation from physics at Planck scale provides a natural way out.

b) Unstable particles with at least two massive particles in the final state of all their decay channels (neutron, nuclei, Δ^{++} , possibly muons and τ's...) become stable at very high energy (Gonzalez-Mestres, 1997a and 1997b). In any case, unstable particles live longer than naively expected with exact Lorentz invariance and, at high enough energy, the effect becomes much stronger than previously estimated for nonlocal models (Anchordoqui, Dova, Gómez Dumm and Lacentre, 1997) ignoring the small violation of relativistic kinematics. Not only particles previously discarded because of their lifetimes can be candidates for the highest-energy cosmic-ray events, but very high-energy cascade development can be modified (for instance, if the π^0 lives longer at energies above $\approx\ 10^{18}\ eV$, thus favoring hadronic interactions and muon pairs and producing less electromagnetic showers).

c) The allowed final-state phase space of two-body collisions is seriously modified at very high energy, especially when, in the vacuum rest frame where expressions (1) - (3) apply, a very high-energy particle collides with a low-energy target (Gonzalez-Mestres, 1997d). Energy conservation reduces the final-state phase space at very high energy and can lead to a sharp fall of cross sections starting at incoming-particle wave vectors well below the inverse of the fundamental length, essentially above $E \approx (E_T\ a^{-2}\ h^2\ c^2)^{1/3}$ where E_T is the energy of the low-energy target. For $a \approx 10^{-33}\ cm$, this scale corresponds to: $\approx 10^{22}\ eV$ if the target is a rest proton; $\approx 10^{21}\ eV$ if it is a rest electron; $\approx 10^{20}\ eV$ for a $\approx 1\ keV$ photon, and $\approx 10^{19}\ eV$ if the target is a visible photon. For a proton impinging on a $\approx 10^{-3}\ eV$ photon from cosmic microwave background radiation, and taking $\alpha \approx 1/12$ as in models based on an isotropic extension of the Bravais lattice dynamics (Gonzalez-Mestres, 1997a), we expect the fall of cross sections to occur above $E \approx 5.10^{18}\ eV$, the critical energy where the derivatives of the mass term $m^2\ c^3\ p^{-1}/2$ and of the deformation term $\alpha\ p\ (k\ a)^2/2$ become equal in the expression relating the proton energy E to its momentum p . With $a \approx 10^{-30}\ cm$, still allowed by cosmic-ray data (Gonzalez-Mestres, 1997d), the critical energy scale can be as low as $E \approx 10^{17}\ eV$; with $a \approx 10^{-35}\ cm$ (still compatible with data in the region of the predicted GZK cutoff), it would be at $E \approx 5.10^{19}\ eV$. Similar considerations lead to a fall of radiation under external forces (e.g. synchrotron radiation) above this energy scale. In the case of a very high-energy γ ray, taking $a \approx 10^{-33}\ cm$, the deformed relativistic kinematics inhibits collisions with $\approx 10^{-3}\ eV$ photons from cosmic background radiation above $E \approx 10^{18}\ eV$, with $\approx 10^{-6}\ eV$ photons above $E \approx 10^{17}\ eV$ and with $\approx 10^{-9}\ eV$ photons above $E \approx 10^{16}\ eV$. Taking $a \approx 10^{-30}\ cm$ would lower these critical energies by a factor 100 according to the previous formulae, whereas the choice $a \approx 10^{-35}\ cm$ would raise them by a factor of 20 . Obviously, any relevant high-energy astrophysical calculation must consider the possibility that Lorentz symmetry be violated at some fundamental length scale.

d) In astrophysical processes, the new kinematics may inhibit phenomena such as GZK-like cutoffs, photodisintegration of nuclei, decays, radiation emission under external forces (similar to a collision with a very low-energy target), momentum loss (which at very high energy does not imply deceleration) through collisions, production of lower-energy secondaries... **potentially solving all the basic problems raised by the highest-energy cosmic rays** (Gonzalez-Mestres, 1997c and 1997d). Due to the fall of cross sections, energy losses become much weaker than expected with relativistic kinematics and astrophysical particles can be pushed to much higher energies (once energies above $10^{17}\ eV$ have been reached through conventional mechanisms, synchrotron radiation and collisions with ambient radiation may start to be inhibited by the new kinematics); similarly, astrophysical particles will be able to propagate to much longer astrophysical distances, and many more sources (in practically all the presently observable Universe) can produce very high-energy cosmic rays reaching the earth; as particle lifetimes are much longer, new possibilities arise for the nature of these cosmic rays. The same considerations apply to nuclei, but their composite nature requires a specific discussion. Models of very high-energy astrophysical processes cannot ignore a possible Lorentz symmetry violation at Planck scale, in which case observable effects are predicted for the highest-energy detected particles.

e) If the new kinematics can explain the existence of $\approx 10^{20}\ eV$ events, it also predicts that, above some higher energy scale (around $\approx 10^{22}\ eV$ for $a \approx 10^{-33}\ cm$), the fall of cross sections will prevent the cosmic ray from depositing most of its energy in the atmosphere (Gonzalez-Mestres, 1997d). Such extremely high-energy particles will produce atypical events of apparently much lower energy. New analysis of data and experimental designs are required to explore this possibility.

Velocity reaches its maximum at $k \approx (8\pi^2\ \alpha^{-1})^{1/4}\ (m\ c\ h^{-1}\ a^{-1})^{1/2}$. Above this value, increase of momentum amounts to deceleration. In our ansatz, observable effects of local Lorentz invariance breaking arise, at leading level, well below the critical wavelength scale a^{-1} due to the fact that, contrary to previous models (f.i. Rédei, 1967), we directly apply non-locality to particle propagators and not only to the interaction hamiltonian. In contrast with previous patterns (f.i. Blokhintsev, 1966), $s - t - u$ kinematics ceases to make sense and the motion of the global system with respect to the vacuum rest frame plays a crucial role. The physics of elastic two-body scattering will depend on five kinematical variables. Noncausal dispersion relations (Blokhintsev and Kolerov, 1964) should be reconsidered, taking into account the departure from relativistic kinematics. As previously stressed (Gonzalez-Mestres, 1997a) , this apparent nonlocality may actually reflect the existence of superluminal sectors of matter (Gonzalez-Mestres, 1996) where causality would hold at the superluminal level (Gonzalez-Mestres, 1997e). Indeed, electromagnetism appears as a nonlocal interaction in the Bravais model of phonon dynamics, due to the fact that electromagnetic signals propagate much faster than lattice vibrations.

In this note, we would like to discuss the consequences of the new kinematics for very high-energy nuclear physics, focusing on the application of the deformed dispersion relation to situations where the particle (proton, nucleus...) is actually a composite object. As compared to a previous attempt to extend to composite and macroscopic objects a different version of deformed special relativity (Bacry, 1993), our paper focuses more on the understanding of the physical content of the kinematics instead of trying to directly extend to all bodies the same underlying deformed Poincaré algebra.

2 Deformed kinematics and composite objects

A basic question is how deformed relativistic kinematics applies to composite objects. By "elementary" particles we should in principle mean those that are generated by fundamental dynamics at Planck scale or beyond this scale and which, at this level, can be described similar to phonons in a lattice (Gonzalez-Mestres, 1997a) or to solitons or topological defects in a medium. This can be the case of quarks, leptons and gauge bosons or, instead, of same preons which would be the constituents of these particles. Because of quark confinement, it can also be the case of hadrons. Supersymmetric extensions of these criteria are obvious. A natural hypothesis, that we have used in previous calculations, may be that c and α have universal values for all "elementary" particles and also for nuclei. But, as we shall see below, the situation for hadrons appears to be less trivial than suggested by this naive formulation and the hypothesis may turn out to be fundamentally wrong for heavy nuclei. The transition from nucleons to nuclei will be a crucial test for all models.

Standard relativity extends trivially from microscopic to macroscopic objects, from constituents to composite entities. When boosting any physical body initially at rest, its rest energy is multiplied by the Lorentz factor $\gamma = (1 - v^2c^{-2})^{-1/2}$ for a velocity $\vec{v}$ and the momentum is given by the galilean momentum times γ . This operation is additive with respect to the constituents, and works in the same way for elementary and composite physical systems. Schematically, relativistic composite models admit a "parton" description where each constituent carries a fraction of the total energy and momentum, and this energy-independent fraction (neglecting radiative corrections) can vary according to a probability distribution given by the basic dynamics. Formally, we can split the rest energy $M\ c^2$ of the composite system in N parts $M_i\ c^2$ ($i = 1, ..., N$, $M_1 + ... + M_N = M$) and attribute to each part: a) an energy $E_i = M_i\ c^2\ \gamma$; b) a momentum $\vec{p_i} = M_i\ \vec{v}\ \gamma$. The fraction of energy and momentum carried by the i-th parton is M_i/M . Then, the speed $v = dE/dp$ is independent of the fraction of the total momentum and energy carried by the parton: all partons can have the same velocity, equal to that of the composite model, independently of how they share the total energy and momentum. This property, essential to the stability of the composite system, is due to the absence of any distance or mass scale other than the total mass (the formal "rest energies" of the partons being fractions of the total rest energy). The deformation of special relativity modifies this situation at very high energy by the introduction of the fundamental scale a associated to nonlocal interactions, and produces leading effects well below $E \approx (2\pi\ a)^{-1}\ h\ c$.

At very high energy, the speed of an "elementary" particle in deformed relativity is given by (Gonzalez-Mestres, 1997a):

$$v = c\ \left[de\ (k\ a)\ /\ d(k\ a)\right] \tag{4}$$

which, for $e\ (k\ a) = [sin^2\ (k\ a) + (2\pi\ a)^2\ h^{-2}\ m^2\ c^2]^{1/2}$ and $m \neq 0$, falls to zero as k approaches the value $\pm\pi/2a$. The velocity vector changes sign at these values of k and tends again to zero as k approaches $\pm\pi/a$. Similar properties, with slightly different formulae, are found in most nonlocal models. A basic property of deformed relativistic kinematics is that: a) the velocity of each parton depends on the fractions it carries of the total energy and momentum; b) the fractions of the total energy and momentum carried by a given parton are not exactly identical; c) for a given value of the total momentum, the total energy depends on the way momentum is shared by the partons. At TeV energies, and taking a close to Planck scale, these effects are very small and can be compensated by very small deviations from the standard parton picture. However, at the highest energies reached by cosmic-ray events, the fluctuations of the total energy due to the combination of standard structure functions with deformed relativistic kinematics for partons would become of the same order as the mass term of the composite system and produce therefore a leading effect. At such energies, we do not expect the parton model to remain valid. With respect to standard relativity, a fundamental change has been operated by the presence of an energy scale (the Planck scale, or any other similar scale) other than the total rest energy and whose effect does not amount to a simple redefinition of partons. The new physics manifests itself at wavelengths well below the inverse of the fundamental length scale, and can be experimentally checked at the highest observed cosmic-ray energies.

Assume that, inside a fast proton, quarks can be considered as "almost-free" parton constituents to which deformed relativistic kinematics can be individually applied. At wave vectors above $k \approx (8\pi^2 \alpha^{-1})^{1/4} (m\ c\ h^{-1}\ a^{-1})^{1/2}$ in the vacuum rest frame, the velocity decreases as momentum increases and the term $\alpha\ (k\ a)^3/2$ in (3) dominates over the term $(2\pi\ a)^2\ h^{-2}\ m^2\ c^2$; no compensation is possible between the derivatives of the two terms. At wave vectors well below $k \approx (8\pi^2 \alpha^{-1})^{1/4} (m\ c\ h^{-1}\ a^{-1})^{1/2}$, it would still be possible to compensate the small effect of the term $\alpha\ (k\ a)^3/2$ in (3) by small corrections to the parton description: in order to preserve the collective propagation of the bound state, the fractions of the total energy and momentum carried by each parton would become slightly velocity-dependent; in order to make the total energy independent of the way momentum is shared by the partons, a momentum-dependent potential term should be added. Above $k \approx (8\pi^2 \alpha^{-1})^{1/4} (m\ c\ h^{-1}\ a^{-1})^{1/2}$, the new kinematics produces a leading effect compelling all partons to have almost the same energy and momentum. Then, in a constituent model and due to the term $\alpha\ (k\ a)^3/2$ which explicitly incorporates the fundamental scale a as the distance (or energy, or mass) scale, the constituents considered as "almost-free" particles would have the same velocity only if they carried almost exactly the same fractions of the total momentum and energy. This definitely forces a departure from the parton model and, since quarks cannot oscillate outside hadrons, it suggests that in a stable hadron (where quarks are confined) there exists only one motion (that of the overall system, instead of a set of "almost-free" parton constituents) to which the deformed quantum relativistic kinematics can be applied. Therefore, it indeed seems natural to extend to such systems the same formulae of deformed relativistic kinematics as for leptons and gauge bosons, with the same values of c and α . Simultaneously, we expect any parton model for hadrons to fail at energies above $\approx\ 10^{19}\ eV$ if $a\ \approx 10^{-33}\ cm$, $\approx\ 10^{20}\ eV$ for $a\ \approx 10^{-35}\ cm$ and $\approx\ 3.10^{17}\ eV$ for $a\ \approx 10^{-30}\ cm$. However, as will be seen below, soliton or bound state dynamics can introduce deviations from the universality of α .

It must also be realized that, in a naive constituent picture, deformed relativistic kinematics with universal values of c and α cannot be applied simultaneously to the constituents and to the composite system. If this were the case, the former could by no means have the same velocity as the latter at wave vectors above $k \approx (8\pi^2 \alpha^{-1})^{1/4} (m\ c\ h^{-1}\ a^{-1})^{1/2}$. If deformed relativistic kinematics were to be applied simultaneously to the constituents and to the composite system, the universality of α would have to be abandoned and a choice of α around $\alpha_C \approx N^{-2} \alpha_P$, where α_P is the value of α for a single constituent and N is the number of constituents, would be required for the composite system. Nevertheless, such a choice seems to lack motivation for hadrons, as quarks are confined and in deformed relativistic kinematics the internal structure of the composite system at very high energy does not allow for several independent, large longitudinal momenta.

Similar questions can be heuristically considered from the point of view of Lorentz contraction and dilation, using simplified soliton models. For instance, in the model based on an analogy with the one-dimensional Bravais lattice (Gonzalez-Mestres, 1997a), we can write the formal expansion (see also Gonzalez-Mestres, 1997f):

$$\phi\,(x \pm a) - \phi\,(x) = \Sigma_{n=0}^{\infty}\, \phi^{(n)}\,(x)\;(\pm a)^n\,(n!)^{-1} \tag{5}$$

where $\phi^{(n)} = d^n\phi/dx^n$ and $\phi\,(x)$ is the fixed-time wave function (extended from integer $x\,a^{-1}$ to real x), so that the expression $\phi\,(x) - [\phi\,(x + a) + \phi\,(x - a)]/2$ is equal to the sum of the even terms (even powers of a) in the right-hand side of (5). Solving the equations of motion by a recurrent procedure starting from the local case, would lead to an expansion:

$$\phi\,(x) = \Sigma_{m=0}^{\infty}\, \phi_m\,(x)\, a^{2m} \tag{6}$$

which is valid for any nonlocal model, whether or not it derives from a lattice. $\phi_0\,(x)$ corresponds to the local, relativistic limit with exact Lorentz symmetry. If the model has solitons, in the relativistic limit their size will be $\gamma^{-1}\,\Delta$, where Δ is a characteristic distance scale from soliton dynamics which cannot be generated from just a and the speed of light c. Such a scale can be provided by a local potential $V\,(\phi)$ which would need a constant with space (or time) dimensionality in order to be compared with the second time derivative and with the space finite difference (or with the second space derivative, in the continuum limit). Then, a dimensionless parameter for power expansion would be: $\xi = \alpha\,(a\,\gamma_R)^2\,\Delta^{-2}$ where α is basically the same coefficient as before, accounting for the coefficients of the power expansion, and γ_R is the standard relativistic Lorentz factor. If the effective inverse squared Lorentz factor γ^{-2} is corrected by a power series of ξ ($\gamma^{-2} = \gamma_R^{-2} + \gamma'\,\xi + ...$, γ' being a constant of order 1), we expect the departure from standard relativity to play a leading role at energies above that for which $\gamma_R^{-2} \approx \alpha\,(a\,\gamma_R)^2\,\Delta^{-2}$, i.e. above $E \approx m\,c^2\,\alpha^{\,1/4}\,(a\,\Delta^{-1})^{-1/2}$. Taking the values $\alpha \approx 0.1$, $m \approx 1\ GeV/c^2$ and $\Delta \approx 10^{-13}\ cm$, this energy scale corresponds to E above $\approx 2.10^{19}\ eV$ for $a \approx 10^{-33}\ cm$, $2.10^{20}\ eV$ for $a \approx 10^{-35}\ cm$ and $\approx 5.10^{17}\ eV$ for $a \approx 10^{-30}\ cm$. For comparison, the maximum velocity in the previous deformed relativistic kinematics occurs at $E \approx 10^{19}\ eV$ for $a \approx 10^{-33}\ cm$, $\approx 10^{20}\ eV$ for $a \approx 10^{-35}\ cm$ and $\approx 3.10^{17}\ eV$ for $a \approx 10^{-30}\ cm$. The departure from standard relativity, concerning the size and internal time scale of the soliton in the vacuum rest frame, indicates that its internal structure is modified and we do not expect the usual parton model to hold any longer.

A rough, indicative illustration of the basic mechanism we just described can be obtained by introducing a small perturbation in the Φ^4 soliton theory. Starting from the equation:

$$c^{-2}\ \partial^2\psi/\partial t^2 \ - \ \partial^2\psi/\partial x^2 \ = \ 2\ \Delta^{-2}\ \psi\ (1\ -\ \psi^2) \tag{7}$$

where Δ is the distance scale characterizing the soliton size (x = space coordinate, t = time coordinate), and writing down the one-soliton solution of this equation:

$$\psi(x\ ,\ t) \ = \ \Phi(y) \ = \ tanh\ (\lambda_0\ y) \tag{8}$$

where $y \ = \ x \ - v\ t$, v is the speed of the soliton, $\lambda_0 \ = \ \Delta^{-1}\ \gamma_R$ and, as before, $\gamma_R \ = \ (1\ -\ v^2\ c^{-2})^{-1/2}$, we can introduce a perturbation to the system by adding to the left-hand side of (7) a term $-\ (a^2/12)\ \partial^4\psi/\partial\ x^4$. The new differential term in the equation corresponds to the term proportional to a^4 in the expansion (5) , and will be compensated at the first order in the perturbation by the replacement:

$$\Phi \ \rightarrow \ \Phi\ +\ \epsilon\ \Phi\ (1\ -\ \Phi^2) \tag{9}$$

where $\epsilon\ \propto\ a^2$. We furthermore replace λ_0 by a new coefficient λ to be determined form the perturbed equation. To first order in the perturbation, we get the solutions:

$$\epsilon \ \simeq \ 1\ -\ \lambda^2\ \gamma_R^{-2}\ \Delta^2 \tag{10}$$

$$\lambda^2 \ \simeq \ [3\ \pm\ (1\ -\ 4\ a^2\ \Delta^{-2}\ \gamma_R^4/3)^{1/2}]\ (1\ +\ a^2\ \Delta^{-2}\ \gamma_R^4/6)^{-1}\ \Delta^{-2}\ \gamma_R^2/4 \tag{11}$$

leading for $\epsilon\ \ll\ 1$ to $(\Delta\ \lambda)^{-2}\ \simeq\ \gamma_R^{-2}\ +\ a^2\ \Delta^{-2}\ \gamma_R^2/3$ in agreement with our previous discussion. Maximum values of v and λ would occur at $\gamma_R \ = \ 2^{-1/2}\ 3^{1/4}\ (\Delta\ a^{-1})^{1/2}$, but our approximations do not apply at this value of γ_R . Deformed kinematics can be obtained at the lowest order in the perturbation. A simplified calculation, valid for a wide class of soliton models, could be as follows. At the first order in the perturbation with respect to special relativity, we start from an effective lagrangian for soliton kinematics:

$$L \ = \ -\ m\ c^2\ \gamma_R^{-1}\ (1\ -\ \rho\ \gamma_R^4) \tag{12}$$

where ρ is a constant proportional to $a^2\ \Delta^{-2}$, according to (10) and (11). From this lagrangian, we derive the expression for the generalized momentum:

$$p \ = \ m\ \gamma_R\ v\ (1\ +\ 3\ \rho\ \gamma_R^4) \tag{13}$$

from which we can build the hamiltonian:

$$H \ = \ p\ v\ -\ L \ = \ m\ c^2\ (\gamma_R\ +\ 3\ \rho\ v^2\ c^{-2}\ \gamma_R^5\ -\ \rho\ \gamma_R^3) \tag{14}$$

which leads to a deformed relativistic kinematics defined by the relation:

$$E\ -\ p\ c \ = \ m\ c^2\ \gamma_R^{-1}\ (1\ +\ v\ c^{-1})^{-1}\ -\ \rho\ \ m\ c^2\ [3\ v\ c^{-1}\ (1\ +\ v\ c^{-1})^{-1}\ +\ 1]\ \gamma_R^3 \tag{15}$$

and, when expressed in terms of p at $v \simeq c$, can be approximated by:

$$E - p\,c \simeq m\,c^2\,(2\,p)^{-1} - 5\,\rho\,p^3\,(2\,m^2\,c)^{-1} \tag{16}$$

where the deformation term $5\,\rho\,p^3\,(2\,m^2\,c)^{-1}$ differs from that obtained from phonon mechanics only by a constant factor $\eta \propto 2\,h^2\,(2\pi\,m\,c\,\Delta)^{-2}$. This factor can be close to 1 only if the mass of the extended system is directly related to its size through a quantum uncertainty relation. Such a situation corresponds to reality for hadrons, but not for nuclei (or larger bodies) where both the mass and the size increase simultaneously as the system gets larger. In general, there seems to be no fundamental reason for η to be exactly 1 , but we have no indication that it should vary like the inverse square of the number of quarks forming the hadron, as previously foreseen for composite models (Gonzalez-Mestres, 1997c). The deformation of relativistic kinematics seems to be driven by soliton dynamics and not by the superposition of deformed kinematics for free quarks. ¿From the above heuristic example, both $\eta > 1$ and $\eta < 1$ are possible but η seems to naturally be ≈ 1 for hadrons and $\ll 1$ for heavy nuclei. Also, the value of α will a priori be different for different hadrons. A more precise estimate of the effect of non-locality in hadronic physics could be obtained by putting QCD on a lattice whose spacing would be the fundamental length a but, due to the very large $\Delta\,/a$ ratio, the lattice size would be far beyond numerical computation potentialities. We shall not attempt here to introduce nonlocality in other theoretical approaches to the structure of hadrons. Looking at the low-speed limit of (15), we find a renormalization of the critical speed parameter c , δc , such that $\delta c\,c^{-1} \approx (\Delta/a)^{-2} \approx 10^{-40}$ for hadrons if $a \approx 10^{-33}\,cm$ and $\Delta \approx 10^{-13}\,cm$. This effect is much smaller than the effects contemplated by other authors (e.g. Coleman and Glashow, 1997) and cannot be excluded by existing data.

Above the critical momentum scale (in the vacuum rest frame) defined by the previous arguments, we can imagine two scenarios for very high-energy hadrons:

- a) As the deformation term $\alpha\,(k\,a)^3/2$ in (3) becomes much larger than the mass term $(2\pi\,a)^2\,h^{-2}\,m^2\,c^2$, the internal structure of the composite system evolves and the soliton may no longer be stable. For hadrons, this would lead to a scenario with free quarks and gluons at high enough energies in vacuum rest frame (kinematic deconfinement). Quarks and gluons with energies just above the threshold for kinematic deconfinement would not be able to transfer energy to slower particles, but could form with them fast aggregates.

- b) Confinement persists, and the departure from standard relativity becomes stronger as momentum increases. As k becomes $\approx a^{-1}$, the soliton may slower down and possibly recover a size similar to its low-momentum rest size. The dynamics well above the critical momentum scale (the scale where the deformation term becomes of the same order as the mass term) is likely to be model-dependent, and no general conclusion can *a priori* be drawn in the lack of a precise dynamical description of hadrons. The assumption that has the same value for leptons, gauge bosons and hadrons is possibly a qualitatively reasonable hypothesis, but not the exact solution of the problem.

In the case of nuclei, where nucleons are not confined but just bound by strong interactions, the question of stability and internal structure of nuclei at very high energy is indeed a crucial one. It can be checked that relative corrections to the boosted transverse energy are proportional to $\approx \alpha\ (k\ a)^2$ and remain very small as long as $k\ a \ll 1$. If, inside the nuclei, the nucleons must be considered as individual particles, the value of α for a given nucleus must be corrected by a factor N^{-2} , where N is now the number of nucleons. If the above description based on soliton models applies to nuclei, and taking Δ to be the same as for single nucleons (the nucleus would be a multi-soliton bound state), the correction factor would be proportional to m^{-2} ($m =$ mass of the nucleus). With $a \approx 10^{-33}\ cm$, and assuming spontaneous disintegration of the nucleus at rest to be protected by a binding energy $\approx 10^{-2}\ m\ c^2$, this version of nuclear deformed relativistic kinematics would still be able to inhibit photodisintegration by cosmic background photons at energies above $\approx N^{2/3}\ 10^{19}\ eV$ if $\alpha \approx 0.1$ for protons. However, if $\alpha = \alpha_{el}$ for elementary particles and $\alpha \approx \alpha_{el}\ N^{-2}$ for a nucleus made of N nucleons, a maximum energy attainable by the nucleus is $E \approx 2\ a^{-1/2}\ m^{1/2}\ \alpha_{el}^{-1/4}\ c^{3/2}\ h^{1/2}\ (2\ \pi)^{-1/2}$, i.e. $E \approx N^{1/2}\ 10^{19}\ eV$ for $\alpha_{el} \approx 0.1$ and $a \approx 10^{-33}\ cm$. Above these energies, the nucleus can spontaneously radiate very high-energy photons. With the same values for α_{el} and a , synchrotron radiation would be inhibited only above $E \approx a^{-1/2}\ m^{1/2}\ \alpha_{el}^{-1/4}\ c^{3/2}\ h^{1/2}\ (2\ \pi)^{-1/2}$, i.e. above $E \approx 0.5\ N^{1/2}\ 10^{19}\ eV$. The physics outcome for nuclei in the GZK cutoff region would therefore be sensitive to the details of nuclear parameters, source distances to earth and acceleration mechanisms. Contrary to our previous estimates based on a universal value of α for elementary particles and nuclei (Gonzalez-Mestres, 1997a and 1997b), unstable nuclei would not necessarily become stable at very high energy; possible departures from standard relativistic lifetimes (if any) will occur only at higher energies, and will be very sensitive to the details of deformed relativistic kinematics.

Another possibility would be to consider the nucleus as a single particle with a single collective motion to which deformed relativistic kinematics applies with the same value of α as for leptons, gauge bosons and hadrons. Indeed, for the nucleus to be stable, all nucleons should propagate at exactly the same speed and therefore cannot be considered at very high energy as individual constituents carrying variable fractions of energy and momentum (even within the limits set by the strength of the interaction, the potential and kinetic energies of nucleons inside a nucleus at rest being in any case much smaller than the nucleon rest energies). Again, only the overall system undergoes an independent, fast motion. Then, the calculations presented in previous papers (e.g. Gonzalez-Mestres, 1997a and 1997b) would apply without further modifications. In particular, fission and decays changing the nature of the nucleus (α , β ...) would be forbidden above well-defined critical energies. A natural prediction of this scenario would be the existence, at very high momentum, of very heavy nuclei impossible to produce in the laboratory. Furthermore, besides the expected fall of elastic, multiparticle and total cross sections (Gonzalez-Mestres, 1997d), the new kinematics may give rise to fundamentally new kinds of reactions. If the value of α is universal for all particles and nuclei, a very high-energy nucleon or nucleus can fusion with a low-energy nuclear target producing a heavier nucleus which would be stable at this value of momentum. At high enough energy, this reaction, which would release an amount of energy of the order of the energy of the target (including its rest energy), can happen several times, leading eventually to a very heavy stable nucleus-like system not described in standard nuclear tables. Contrary to quark nuggets, these objects would in general be unstable at low energy and cannot be found at nonrelativistic speeds. Experiment seems thus, in principle, to be able to discriminate between different scenarios for very high-energy nuclear physics, in particular concerning the crucial question of the values of α for hadrons and nuclei as compared to leptons and gauge bosons.

To try to settle the discussion, it seems relevant to consider extrapolations to atoms, molecules and macroscopic systems. It is not obvious that such extrapolations exist, and that such objects can be accelerated with respect to the vacuum rest frame (without being disintegrated) to speeds close to the above mentioned maximum, or that they can reach the kinematical region where the deformation of relativistic kinematics dominates over the mass term. If this is the case, there is no compelling argument to apply to these systems the same deformed kinematics as to "elementary" particles, hadrons or nuclei. The size of and atom being much larger than that of a nucleus, and the binding energy much lower, it may seem reasonable to consider the atomic nucleus and the electrons as separate particles to which deformed relativistic kinematics applies individually. This seems also to follow from consistency requirements: a body with mass $10^{25}\ eV\ c^{-2}$ ($\approx 10^{-8}\ gm$) moving at galactic speed would most likely have with respect to the vacuum rest frame (e.g. assuming it to be close to that defined by cosmic microwave background radiation) a momentum $\approx 10^{22}\ eV\ c^{-1}$. It would obviously be erroneous to apply single-particle deformed relativistic kinematics to such an object with the same values of α as for elementary particles. At this stage, two different simplified approaches can be considered:

- *Model i)* . Due to the very large size of atoms, as compared to nuclei, the transition from nuclear to atomic scale appears as a reasonable point to stop considering systems as "elementary" from the point of view of deformed relativity. α would then have a universal value for nuclei and simpler objects, but not for atoms and larger bodies.

- *Model ii)* . The example with Φ^4 solitons suggests that hadrons can have values of α close to that of leptons and gauge bosons, and the transition may happen continuously at fermi scale, when going from nucleons to nuclei. Then, the value of α would be universal (or close to it) for leptons, gauge bosons and hadrons but follow a m^{-2} law for nuclei and heavier systems, the nucleon mass setting the scale.

Experimental tests should be performed and equivalent dynamical systems should be studied. However, *Model i)* would lack a well-defined criterium to separate systems to which the deformed relativity applies with the same value of α as for leptons and gauge bosons from those to which this kinematics cannot be applied, and to characterize the transition between the two regimes. The above obtained $m^{-2}\ \Delta^{-2}$ dependence of the coefficient of the deformation term for extended objects, as described in *Model ii)*, seems to provide a continuous transition from nucleons to heavier systems, naturally filling this gap. On the other hand, a closer analysis reveals that there is indeed a discontinuity between nuclei and atoms, as foreseen in *Model i)* . As long as the deformation term in electron kinematics can be neglected as compared to the electron mass term, we can consider that most of the momentum of an atom is carried by the nucleus and *Model ii)* may provide a reasonable description of reality. But, when the electron mass term becomes small as compared to the part of the energy it would carry in a parton model of the atom, such a description becomes misleading. To have the same speed as a nucleon, the electron must then carry nearly the same energy and momentum. We therefore propose a modified version of *Model ii)* with $\Delta \approx 10^{-13}\ cm$ from hadrons and nuclei where, for atoms and larger neutral systems, the coefficient of the deformation term would be corrected by a factor close to 1 at low momentum and to 4/9 at high momentum if the number of neutrons is equal to that of protons. This model is obviously approximate and should be completed by a detailed dynamical calculation that we shall not attempt here. It assumes that electrically neutral bodies can reach very high energies per unit mass, which is not obvious: spontaneous ionization may occur at speeds (in the vacuum rest frame) for which the deformation term in electron kinematics becomes larger than its mass term.

Then, for bodies heavier than hadrons, the effective value of α would decrease essentially like m^{-2} . Applying a similar mass-dependence to the κ parameter of a different deformed Poincaré algebra considered by previous authors (Bacry, 1993 and references therein), i.e. $\kappa \propto m$ for large bodies, yields the relation:

$$F\ (M_0\ ,\ E_0)\ =\ F\ (M_1\ ,\ E_1)\ +\ F\ (M_2\ ,\ E_2) \tag{17}$$

with:

$$F\ (m\ ,\ E)\ =\ 2\ \kappa\ (m)\ \ sinh\ [2^{-1}\ \kappa^{-1}\ (m)\ E] \tag{18}$$

where $M = M_1 + M_2$, $\kappa(m)$ is our above mass-dependent version of the κ parameter of the deformed Poincaré algebra used by these authors, and E_0 is the energy of a system with mass M made of two non-interacting subsystems of energies E_1 and E_2 and with masses M_1 and M_2 . Defining mass as an additive parameter, the rest energy $E_{i,rest}$ (in the vacuum rest frame) of particle i ($i = 0\ ,\ 1\ ,\ 2$) is given by the equation:

$$M_i\ c^2\ =\ 2\ \kappa\ (M_i)\ \ sinh\ [2^{-1}\ \kappa^{-1}\ (M_i)\ E_{i,rest}] \tag{19}$$

and tends to $M_i\ c^2$ as $\kappa\ (M_i) \rightarrow \infty$. Equations (17) and (18) lead to additive relations for the energy of macroscopic objects if the proportionality rule $\kappa\ (m) \propto m$ is applied. From our previous discussion with a different deformation scheme, such a choice seems to naturally agree with physical reality. Then, contrary to previous claims (Bacry, 1993; Fernandez, 1996), the rest energies of large systems would be additive and no macroscopic effect on the total mass of the Universe would be expected.

3 Experimental considerations

Definitely, the main and most fundamental physics outcome of very high-energy cosmic-ray experiments involving particles and nuclei may eventually be the test of special relativity. If Lorentz symmetry is violated at Planck scale, the highest-energy cosmic ray events may, if analyzed closely and with the expected high statistics from future experiments, provide a detailed check of different models of deformed relativistic kinematics. If some of the highest-energy cosmic rays are nuclei, their study would yield crucial information on Planck-scale corrections to the kinematics of composite objects. If Lorentz symmetry turns out to be violated, the comparison between the properties of nucleons and those of different nuclei would be crucial to understand how deformed Poincaré algebras possibly apply to matter. Indeed, a detailed fit to data would involve and check all the relevant parameters. Although the models we consider predict the new phenomena to occur in all cases at energies above $\approx 10^{17}\ eV$ for hadrons and nuclei, extrapolation over several orders of magnitude is involved and many theoretical uncertainties remain. Furthermore, until now we have been interested only in observable effects at leading level, but small deviations from standard relativistic values of physical parameters may also be detectable (although, in the models we considered, the energy dependence of the predicted new effects is always very sharp since a fourth power of energy is often involved in the calculations). Therefore, it would be most crucial to find an efficient way to test special relativity for cosmic rays in the $10^{16} - 10^{17}\ eV$ range with the help of LHC using the potentialities of the FELIX experiment (e.g. Eggert, Jones and Taylor, 1997). Such a test is actually compulsory before trying to consistently compare LHC and cosmic-ray data for other practical purposes.

Lorentz symmetry violation prevents naive extrapolations from reactions between two particles with equal, opposite momenta in the vacuum rest frame (similar to colliders) to reactions where the target is at rest in this frame (similar to cosmic-ray events). *Assuming the earth to move slowly with respect to the vacuum rest frame* (for instance, if the "absolute" frame is close to that defined by the requirement of cosmic microwave background isotropy), the described kinematics leads, at the highest observed cosmic ray energies, to a departure from the parton model that can strongly influence cascade development at very high energy. Other effects, such as a longer π^0 lifetime as compared to standard relativity or the possibility that the primary hadron be a neutron, would increase the interest and originality of very high-energy showers. Furthermore, we predict (Gonzalez-Mestres, 1997d) the existence of a maximum energy deposition for high-energy cosmic rays in the atmosphere, in the rock or in a given underground or underwater detector. Well below Planck energy, a very high-energy cosmic ray would not necessarily deposit most of its energy in the atmosphere: its energy deposition decreases for energies above a transition scale, far below the energy scale associated to the fundamental length. The maximum allowed momentum transfer in a single collision occurs at an energy just below $E_{lim} \approx (2\pi\ )^{-2/3}\ \alpha^{-1/3}\ (E_T\ a^{-2}\ h^2\ c^2)^{1/3}$, where E_T is the energy of the target in the vacuum rest frame. For E above E_{lim} ($\approx 4.10^{22}\ eV$ if the target is a non-relativistic nitrogen or oxigen nucleus, $\alpha \approx 0.1$ for the cosmic ray and $a \approx 10^{-33}$ cm), the allowed longitudinal momentum transfer falls, typically, like p^{-2} (obtained differentiating the term $\alpha \, k^2 \, a^2 \, p \, c/2$). At energies around E_{lim} , the cosmic ray will in our scenario undergo several scatterings in the atmosphere and still lose there most of its energy, possibly leading to unconventional longitudinal cascade development profiles that could be studied in detail by very large-surface air shower detectors like the AUGER observatory (AUGER Collaboration, 1997). Above E_{lim} , the cosmic ray can indeed cross the atmosphere keeping most of its momentum and energy and deposit its energy in the rock or in water, or possibly reach and underground or underwater detector. Thus, some unconventional cosmic-ray events of apparent energy far below 10^{20} eV , as seen by earth-surface (e.g. air shower), underground or underwater detectors, may actually be originated by extremely high-energy cosmic rays well above this energy scale but interacting weakly because of deformed kinematics.

Nuclei would play a crucial role in the analysis of these phenomena, due to their sensitivity (through the values of the α parameter) to the basic mechanisms of Lorentz symmetry violation and (hopefully) to the possibility to explore different nuclear masses in a high-statistics experiment devoted to the highest-energy cosmic rays.

Constraints on the fundamental length a were derived (Gonzalez-Mestres, 1997d) assuming deformed relativistic kinematics with universal values of α and c , and full-strength Lorentz symmetry violation at the fundamental length scale. In spite of the criticism presented in this paper, such estimates remain useful. The combined absence of GZK cutoff and existence of $\approx 10^{20}$ eV energy deposition from cosmic rays in the atmosphere lead to a in the range 10^{-35} $cm < a < 10^{-30}$ cm (energy scale between 10^{16} and 10^{21} GeV). The lower bound comes from the requirement that the violation of local Lorentz invariance at the fundamental length scale be able to influence particle interactions at the $10^{19} - 10^{20}$ eV energy scale strongly enough to suppress the GZK cutoff. The upper bound is derived from the existence of events with $\approx 10^{20}$ eV energy deposition in the atmosphere (Linsley, 1963; Lawrence, Reid and Watson, 1991; Afanasiev et al., 1995; Bird et al., 1994; Yoshida et al., 1995; Hayashida et al., 1997). Then, the departure from the parton model is expected to occur at an energy scale between $\approx 3.10^{17}$ eV (for $a \approx 10^{-30}$ cm) and $\approx 10^{20}$ eV (for $a \approx 10^{-35}$ cm), in all cases at energies below the highest measured cosmic ray energies. Assuming a universal value of $\alpha \approx 0.1$ for leptons, gauge bosons and hadrons, and the highest-energy cosmic rays to be nucleons, it would also be possible to fit the data with a model where the GZK phenomenon acts at energies below 10^{20} eV but is inhibited above this energy. The best value of a for this purpose would be $\approx 3.10^{-36}$ cm , slightly below the range considered in our previous estimates. It would lead to a failure of the parton model at energies above $\approx 2.10^{20}$ eV , and to longer π^0 lifetimes (inhibiting electromagnetic showers and favouring strong interactions leading to pairs of muons) above $\approx 2.10^{19}$ eV (this phenomenon occurs above $E \approx 10^{18}$ eV if $a \approx 10^{-33}$ cm). Energy losses in acceleration processes would be inhibited only at higher energies than previously considered, but the phenomenon would still be crucial for the highest-energy cosmic rays. If the hypothesis of full-strength Lorentz symmetry violation at the fundamental length scale is abandoned, the roles of α and a are expressed by a single parameter, $\alpha\ a^2$: then, $a \approx 3.10^{-36}\ cm$ and $\alpha \approx 0.1$ would be equivalent to $a \approx 10^{-33}\ cm$ and $\alpha \approx 10^{-6}$. However, since α is not larger than ≈ 0.1 , a cannot be smaller than $\approx 3.10^{-36}\ cm$. If Lorentz symmetry violation at the fundamental length scale is very weak, this scale may occur well below Planck scale without contradicting data: $a \approx 10^{-33}\ cm$ and $\alpha \approx 0.1$ would be equivalent to $a \approx 10^{-26}\ cm$ (just below length scale associated to the highest cosmic ray energies) and $\alpha \approx 10^{-15}$. Only the detection of cosmic rays with energies above $E \approx 10^{21}\ eV$ would possibly allow to exclude a fundamental length scale $a \approx 10^{-26}\ cm$. Apart from this kind of ambiguity, a detailed analysis of the structure of cosmic-ray events at energies above $\approx 10^{17}\ eV$ would allow to discriminate between different scenarios. Future colliders with energies above LHC energies would allow for direct comparison (check of Lorentz invariance) between collider events and cosmic-ray events in this energy range.

Then, very high-energy accelerator and cosmic-ray experiments would indeed be complementary research lines: the results of both kinds of experiments would not be equivalent up to Lorentz tranformations. A $p - p$ collider at $\approx 700\ TeV$ per beam, accelerating also nuclei, could make possible direct tests of Lorentz symmetry violation, comparing collisions at the accelerator with collisions between a $\approx 10^{21}\ eV$ proton of cosmic origin and a proton or nucleus from the atmosphere (an important step in this direction could be taken by the VLHC machine, Very Large Hadron Collider reaching $\approx 100\ TeV$ per beam, which corresponds to cosmic proton energies $\approx 2.10^{19}\ eV$) . Simultaneously, other kinds of tests may be possible through the lifetimes and decay products of very high-energy unstable particles (Gonzalez-Mestres, 1997a and 1997b) in the cosmic-ray events producing the highest-energy secondaries (e.g. by looking in some way at the first π^0 's produced). We would be confronted to a new situation, contrary to conventional expectations, if the cosmic rays at the highest possible energies interact more and more weakly with matter because of kinematical constraints. The existence of a maximum energy of events generated in the atmosphere would not correspond to a maximum energy of incoming cosmic rays. Unconventional events originated by such particles may have been erroneously interpreted as being associated to cosmic rays of much lower energy. New analysis seem necessary, as well as new experimental designs using perhaps in coincidence very large-surface detectors devoted to interactions in the atmosphere (like AUGER) with very large-volume underground or underwater detectors (like AMANDA) and with balloon or satellite experiments able to closely analyze the early cascade development.

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