Ghost-like states and a new "hidden" indistinguishability principle, beyond the purview of the Pauli exclusion principle

Syed Afsar Abbas
Centre for Theoretical Physics, JMI University, New Delhi-110025, India
(email: drafsarabbas@gmail.com)

Abstract

Indistinguishability of identical particles, manifested through entanglement, besides being essential in particle and nuclear physics, forms the basis of much current research in fields such as, quantum computers, quantum cryptography, quantum teleportation, etc.. However, note that the symmetrization principle, implemented e.g. through the Pauli exclusion principle for fermions, is actually an extra postulate, imposed on what is believed to be the basic and essential quantum mechanics. So one may ask, whether there is an indistinguishabilty principle, which arises within a "pure" quantum mechanical framework. In this paper we show that indeed, this is true. Through a careful study of the quantum mechanics of deuteron, we show that there is a new "hidden" indistinguishability principle. This is indepedent of any entanglement principle or Pauli exclusion principle. This "hidden" indistimguishability is manifested through a "hidden" ghost-like state.

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In recent years, quantum mechanics has once again come into centre stage, mainly because of the ongoing upsurge of demands arising from within the fields of quantum computers, quantum cryptography, quantum teleportation etc. [1] [2]. So many of the issues within quantum mechanics, which had traditionally been relegated to the realm of philosophy, have become very real now. One such issue is that of the role of entanglement and that of the Pauli exclusion principle, which have been instrumental for the present successful theories in physics like, particle physics, nuclear physics etc..

All experiments as of now, offer unambiguos support for quantum mechanics. Much of this is based on the proper application of the basic concept of entanglement, and for fermions, its implementation through the application of the Pauli exclusion principle. In quantum field theory, this is the result of the spin statistics theorem. The basis of entanglement lies in how we treat the identical and indistinguishable particles in quantum mechanics. In this paper, we study this issue within the context of the bound state of deuteron, which happens to be the lightest nucleus. A careful study shows that a new perspective on indistinguishabilty, allows us to develop a consistent quantum mechanical structure, which does not require the concepts of entanglement or the Pauli exclusion principle. This demands existence of ghost-like states.

Most physicists would say that in essence, the study of quatum mechanics lies in trying to unravel the basic meaning, both mathematically and physically, of what is superposition, evolution and measurement of the quantum states [3]. However to include the behaviour of matter in bulk, one requires an extra postulate of a "symmetrization principle". For identical fermions it demands the phenomenological imposition of the Pauli exclusion principle. Today this is the structure of a successful and extended quantum mechanics.

Note that the Pauli exclusion principle has not been proven to be a fundamental property of quantum mechanics [3], [4]. However, it continues to be a very successful phenomenological principle in physics, as made clear by its usefulness in particle physics, nuclear physics etc.. Hence one may ask, if there is no other way of implementing inditistinguishability of identical fermions in quantum mechanics? Most may answer in the affirmative. However, in this paper, we show that indeed, quantum mechanics allows for a narrow window for this possibility to become real. Revisiting Wigner's analysis, of the quantum mechanically bound deuteron in a central potential well [5], we show that, indeed, there is present another "hidden" intrinsic indistinguishabilty, which has unfortunately been missed as of now. This "hidden" indistinguishability principle is shown to be implemented by a "hidden" ghost-like state [6]. Now the concept of a ghost field arises in quantum field thery [6]. However, we find here that it plays a fundamental role in hadron/nucleus, albeit still "hidden" inside it.

Deuteron is a unique nucleus in many ways: (a) The charge-radius in deuteron lies within $\sim 2.1 fm$, while beyond this lies a huge amount of matter extending to over $\sim 4.2 fm$, see Fig. 1 and 2 in Appendix A. This structure is completely different from the charge distribution of all heavier nuclei $A \geq 3$, wherein charge and matter radii are nearly equal to each other. (b) Deuteron is the only nucleus (besides triton H^3 and helion He^3) which has just one bound state with no excited state whatsoever. So it may not be surprising, if the quantum mechanical structure of deuteron, turns to be different from that of all the other nuclei. Let us see.

Interestingly, deuteron is also the only nucleus which can be solved analytically by forming a complete model of interaction between the two particles. Wigner in his pioneering work on deuteron, had taken proton and neutron as distinguishable particles [5] . He took proton and neutron as bound by a suitable quantum mechanical central potential which provided just a single experimental bound state (with no other bound state) and gave the proper experimental radius of deuteron [5] . His analysis even today is well accepted and in unchanged manner, is considered a basic and a good picture of deuteron. Important to re-emphasize that upto this point, Wigner had treated protons and neutrons as distinguishable particles. For the sake of completeness we present a short description of this work in Appendix A.

Next in conventional manner, assuming the ground state of deuteron to be in orbital angular mamentum L=0 state, to match the empirically observed total angular momentum of deuteron to be J=1, one takes the total spin of deuteron to be S=1 [7] . Knowing that proton-proton and neutron-neutron pairs do not form bound states, one traces it to the fact that for the isospin group structure $SU(2)_I$, the bound state of deuteron should correspond to isospin T=0 state. Thus finally one ends up treating proton and neutron as indistinguishable particles through the application of the generalized Pauli exclusion principle. This is the conventional way of saying that deuteron corresponds to T=0 state of the group $SU(2)_I$. Below we show that actually the bound state of deuteron is independent of the group $SU(2)_I$.

But note that the above demand of indistinguishability on proton and neutron, was an addendum, onto the initial distinguishability, as demanded by Wigner in his work. This point is further strengthened by the fact that the total nuclear potential taken to define nuclei, is schematically written as

$$V_T = V_W(r) + \sum V_{others} \tag{1}$$

where the first term is the central potential of Wigner for distinguishable neutron and proton. Also it is the only non-exchange term in V_T. Rest of the terms, lumped together schematically, are several exchange potentials (in various standard forms) and/or other phenomenological terms used by different groups. However the only property which allows them to be lumped together as above, is that in all of these, protons and neutrons are taken as indistinguishable particles. In fact as such, the above separation of the potential on the right, is actually telling us whether the neutron and proton be taken as distinguishable (as in the first term due to Wigner), and the others, where indistinguishabilty of proton and neutron is imposed through the generalized Pauli exclusion principle.

Note that Wigner had further, also used his model of distinguishable proton and neutron to explain the data of proton-neutron scattering [8], using similar central binding potential (see Appendix A). Surprisingly while he had succeeded in giving a good description of the bound state of deuteron, his analysis failed badly to explain the corresponding scattering data of neutron-proton. He realised later that this was due to neglecting the spin degree of freedom, S=1 in his analysis; and then the fit with experimental data was good ([7]; p. 31,32). However his deuteron was still bosonic.

Note that the scattering of proton and neutron demands that these be treatd as indistinguishable particles, and thus correspond to the fundamental representation $\binom{p}{n}$ of the isospin group in $SU(2)_I \otimes SU(2)_S$.

However this is not the same thing as the proton-neutron bound state of deuteron. wherein there appears to be no demand of indistinguihability between neutron and proton, to obtain its primary properties of binding energy and the root mean square radius. These are obtained completely consistently by treating proton and neuttron as being distingishable fermions.

Note that therefore, Wigner's analysis is independent of any isospin entanglement principle and of the Pauli exclusion principle. But we still have the issue of the fact that in all this analysis by Wigner, proton and neutron are distinguishable. Actually there is a subtle point here. We shall show here, that indeed, as external entities, proton and neutron are disntinguishable, but as a bound entity, they are quantum mechanically actually indistinguishable, and which is independent of the isospin entanglement or the Pauli exclusion principle.

So how does quantum mechanics makes this possible? Even in the classical Hamiltonian itself (see Appendix A), the two particle problem has been reduced to the dynamics of a a single fictitious particle with a reduced mass μ. Now even classically, within this single fictitious particle, the original independent particles have lost their identity. You cannot tell which one is which, as separate individual particles within this new fictitouis entity. Now for our quantum mechanical deuteron with $m_p \sim m_n$, within this reduced particle mass entity, clearly the two, proton and neutron too, are loosing their identity. And being microscopic quantum mechanical particles, this loss of identity, means that in reality, they have actually become indistinguishable. This is consistent with the Heisenberg Uncertainty Principle. This indistinguishability, has been difficult to be recognized as such, due to the fact that it is actually "hidden" inside the fictitious-reduced-mass entity. Thus finally, this new "hidden" indistinguishabilty, completely defines the bound state of deuteron.

We re-emphasize the same point, by stating that the two body problem of a proton and neutron interacting with a force given by a potential $V(\vec{r_p} - \vec{r_n})$ reduces to the classical mechanics centre of mass transformation, to the problem of a fictitiuos single particle in a potential well $V(r)$, where the coordinates of the effective single particle r, θ, ϕ, are actually the coordinates of the neutron relative to the proton, with the mass of this "single particle" being the reduced mass μ. But there is an ambiguity, in that it might as well be the coordinates of the proton relative to the neutron. This ambiguity is not removed by any explicit exchange symmetry. This is taken care of by the hidden indistinguishabilty imposed by the fictitious "single particle".

Hence the quatum mechanical study of the bound state of deuteron by Wigner, with suitable improvement in perspective, is found to have a "hidden" indistinguishable proton-neutron pair. This allows us to view it as a complete structure within itself. It does not require any extraneous effects of entanglement as required by the Pauli Exclusion Principle.

Einstein, Podolsky and Rosen's paper was entitled "Can quantum mechanical description of physical reality be considered complete" [9]. Indeed, we emphasize that the improved quantum mechanical solution of the bound state of deuteron as presented here, does indeed correspond to a model which appears to fulfil the denands of the title of Einstein, Podolsky and Rosen paper. This point is further consolidated by the fact that in the paper [9] they state, "If without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of physical quantity, then there exists an element of physical reality corresponding to this quantity." Note that as per eqn. (1) the total potential is the sum of the Wigner term and the other set of exchange terms. Hence to use the standard method of exchange potential (eg. without which we can not discuss the nucleon-nucleon scattering), one is lame without a pre-existing first term of Wigner. However, Wigner's central potential term in itself, a priori, does not make any demands on the pre-existence of the exchange terms. Hence Wigner's potential term is more primitive/fundamental/basic in terms of physical and mathematical reality. And as shown here, this does provide a complete and self-consistent description of the bound state of deuteron. This justifies our claims about the veracity of Einstein, Podolsky and Rosen's statements about quatum mechanics for our model. Hence the "hidden" indistinguishability in deuteron is of "pure" quantum mechanical in nature.

We re-emphasize that the symmetrization principle for indistinguishability, implemented e.g. through the Pauli exclusion principle for fermions, is actually an extra postulate, imposed on what is believed to be the basic and essential structure of quantum mechanics [3]. Through a careful study of the quantum mechanics of deuteron, we have shown that there is a new "hidden" indistinguishability principle. This "hidden" indistinguishability principle, is indepedemt of any entaglement principle or Pauli exclusion principle. Hence this new indistinguishability principle is part of a "pure" quantum mechanical structure.

Next, note that given the fact that $m_p \sim m_n$, and that both partake in the nuclear strong interaction, hence these, though distinguishable, should still display the antisymmetry of neutron-proton scattering system in the isospin T=0 state. So in the "hidden" indistinguishabilty above, we would have expected it to have displayed fermionic character. How is it missing in our picture? This may be explained by the fact that the indistinguishability here is "hidden", and therefore the antisymmetry be "hidden" too. This is reminiscent of the ghost-like states in quantum field theory [6].

In non-abelain gauge theories, there arise fields with wrong statistics, e.g. scalar fields which display fermionic character [6]. However it is possible to project out these ghost fields consistently from the Hilbert spaces of the initial and the final states. This ensures that only ghost-free physical states evolve into other ghost-free physical states. Because of this, the ghost fields appear only inside loops as virtual particles. The ghost fields are threrefore irrelevant for the tree level diagrams.

It is exactly this feature, that the "hidden" indistinguishabilty is providing a "hidden" ghost-like statistics inside the deuteron. Thus as this is "hidden", it was therefore hard to be extracted as such. But it is there to ensure, that deuteron has structure independent from that of the group $SU(2)_I$ and that of the Pauli exclusion principle.

In summary, in the case of Faddeev-Popov ghosts [6], one is forced to include fictitious particles called "ghosts". Although these are spin-0 particles, the mathematical requirement is that these fields be Grassmannn variables. Thus they display fermionic character, thus contradiucting the spin-statistics theorem which applies to all particles.

Now in our case, we have discovered a new hidden indistinguishability principle, and which is outside the purview of the isospin $SU(2)_I$ entanglement and the Pauli exclusion principle. Analogous to the quantum field theory ghost case, we have an extra hidden antisymmetry brought in by the hidden ghost-like state in deuteron. This ensures that Wigner's quantum structure of deuteron has the proper overall antisymmetry, so that it is consistent with the neutron-proton scattering antisymmetry strucure, which arise for the T=0 state of isospin $SU(2)_I$ group.

Appendix A:

Classically the Hamiltonian of a bound system of proton and neutron, is written in terms of two independent motions, one of a fictitious reduced mass μ particle under potential V(r), and a fictitious Centre of Mass entity, as

$$H = \frac{\vec{p_p}^2}{2m_p} + \frac{\vec{p_n}^2}{2m_n} + V(\vec{r_p} - \vec{r_n}) = \frac{\vec{P}^2}{2M} + \frac{\vec{p}^2}{2\mu} + V(r); \vec{r} = \vec{r_p} - \vec{r_n}; \mu = \frac{m_p m_n}{m_p + m_n} \tag{2}$$

$$M = m_p + m_n; \quad \vec{P} = \vec{p_p} + \vec{p_n}; \quad \vec{R} = \frac{m_p \vec{r_p} + m_n \vec{r_n}}{m_p + m_n}; \quad \vec{p} = \frac{m_n \vec{p_p} - m_p \vec{p_n}}{m_p + m_n} \tag{3}$$

The same holds true quantum mechanically as well. As per the left hand side Hamiltonian, we have a two body problem with the corresponding wave function of deuteron as $\Psi = \Psi(\vec{r_p}, r_n)$, which treats the two particles as fundamentally independent entities. Instead of solving this two-body problem, fortunately we have the simpler expression of the right-hand side Hamiltronian in terms of new variables.

The new variables obey the same commutation relations corresponding to separable Centre of Mass, and relative coordinates of two fictitious particles of positions $\vec{r}$ and $\vec{R}$, and momenta $\vec{p}$ and $\vec{P}$ respectively as,

$$[r_k, r_k] = i\hbar\,; \quad [R_k, R_k] = i\hbar\,; \quad (k = x, y, z) \tag{4}$$

Write Hamiltonian and its solution as

$$H = H_R + H_r \quad , \quad \Psi(\vec{R}, r) = \phi_{CM}\; u(r) \tag{5}$$

We work in the Centre of Mass coordinate system, with the wave function u(r) of the fictitiuos particle, satisfying Schroedinger equation

$$-\frac{\hbar^2}{2\mu}\frac{d^2u}{dr^2} + V_W(r)u = eu \tag{6}$$

Assume that potential $V_W(r)$ has a finite range b (see schematic Fig. 1) . If there is only one bound state, it should be a zero orbital angular momentum s-state [5]. We follow similar but simpler analysis done by Brink [7]. As used by Wigner, Fig. 1 shows, (A) a square-well potential and (B) Eckart's potential. In Fig. 2 we show corresponding solutions of the square of the

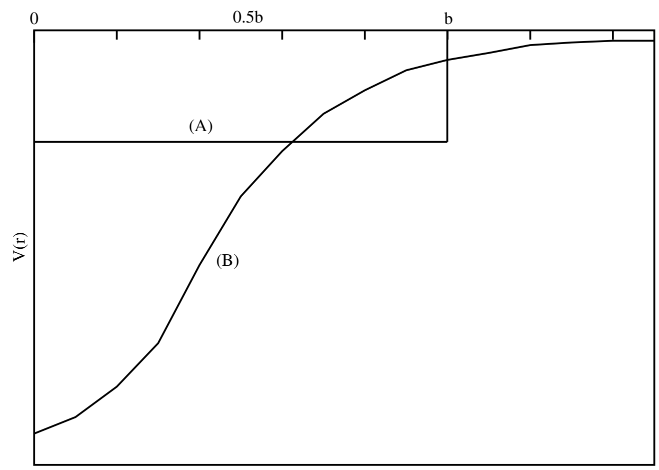

Figure 1: Schematic potentials (A) square-well potential, (B) Eckart's potential

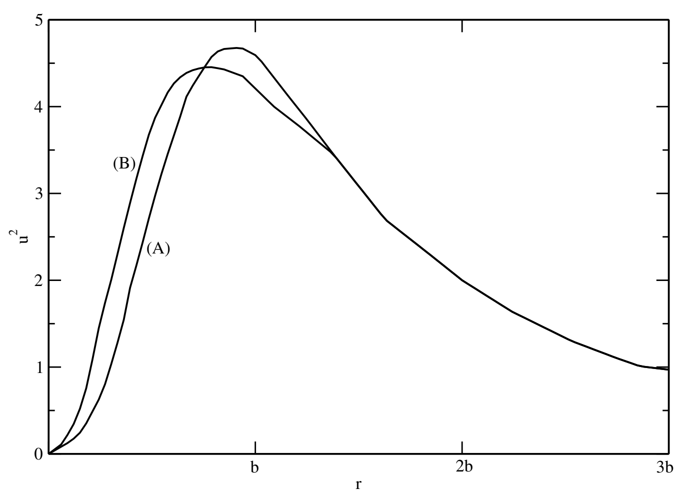

Figure 2: Square of deuteron wave function; (A) square-well potential with range b, (B) range parameter $\rho = \frac{b}{4}$ Eckart's potential [7]

wave function of deuteron. They had used the wave function for $r > b$ as $\sim A\, exp(-\gamma r)$. Through their numerical analysis, they find that for deuteron, the actual value of $\gamma b \sim \frac{1}{3}$, and the probability of finding neutron-proton deuteron wave fonction outside the interaction range (extending way beyond 4.2 fm) is $\sim \frac{3}{4}$. Thus numerically 75% of the wave function of deuteron lies outside the range of the potential. Note that from the electron scattering the charge rms radius of deuteron is 2.1 fm, which is well inside the range of the potentials and as such defines the actual range of the potential b.

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