NLS ground states on graphs

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Abstract

We investigate the existence of ground states for the subcritical NLS energy on metric graphs. In particular, we find out a topological assumption that guarantees the nonexistence of ground states, and give an example in which the assumption is not fulfilled and ground states actually exist. In order to obtain the result, we introduce a new rearrangement technique, adapted to the graph where it applies. Owing to such a technique, the energy level of the rearranged function is improved by conveniently mixing the symmetric and monotone rearrangement procedures.

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1 Introduction

In this paper we investigate the existence of a ground state for the NLS energy functional

$$E(u,\mathcal{G}) = \frac{1}{2}\|u'\|^2_{L^2(\mathcal{G})} - \frac{1}{p}\|u\|^p_{L^p(\mathcal{G})} \tag{1}$$

with the *mass constraint*

$$\|u\|^2_{L^2(\mathcal{G})} \;=\; \mu, \tag{2}$$

where $\mu > 0$ and $p \in (2,6)$ are given numbers and $\mathcal{G}$ is a *connected metric graph.*

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