The Standard Model is a Complete Theory in itself and Quantum Electrodynamics is mathematically and physically a self-consistent theory

Syed Afsar Abbas
Department of Physics
Aligarh Muslim University, Aligarh - 202002, India
(e-mail : drafsarabbas@yahoo.in)

Abstract

Due to the possible existence of the Landau Pole (or Moscow Zero) it is commonly believed that Quantum Electrodynamics is mathematically an inconsistent theory. However for a theory which has been eulogized by Feynman as the "jewel of physics", the above statement is rather unpalatable. Indeed, on the basis of a careful study of the intrinsic structure of the Standard Model, we show here as to how the Landau Pole problem is solved. In Quantum Electrodynamics, we show as to what the bare mass and the bare charge actually are. Now these bare quantities play their proper role within the renormalized mass and the renormalized charge respectively. We shall then demonstrate as to how Quantum Electrodynamics is fully self-consistent both physically and mathematically. This work thus shows that the Standard Model is a Complete Theory in itself and that it is this that generates the QED. Thus QED is a secondary theory. It is therefore shown that all putative extensions for unification beyond the Standard Model, like the GUTs, the Supersymmetric extensions etc are inconsistent with the Standard Model and hence are ruled out.

Keywords: Standard Model, Renormalization

(I) Introduction

Because of the extremely accurate predictions of the physical quantities like the anomalous magnetic moment of the electron and the Lamb shift of the energy levels of the hydrogen atom, that Quantum Electrodynamics has been honoured as "the jewel of physics" by Feynman [1]. However, shocking and extremely upsetting was the fact that QED also had the property that the interactions become infinitely strong at short enough distance scales. Such a phenomenon called the Landau Pole (or Moscow Zero) made it plausible that QED was inconsistent. Perturbed by this Feynman wrote [1], " The shell game that we play .... is technically called renormalization. But no matter how clever the word, it is still, what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of QED is mathemtically consistent. It is surprising that the theory still hasn't been proved self-consistent one way or the other by now. I suspect that renormalization is not mathematically legitimate". Though this statement was written around 1985, the issue as to the possible inconsistency of QED remains the same today.

In this paper, we show that actually QED is physically and mathematically self-consistent. Ironically, in contrast to Feynman's point of view, to do the above, we do require the complete machinery of the quantum field theory. That is that QED is not inconsistent but this is due to intrinsic use of the property of renomalizability of the Standard Model.

It will be shown that the Standard Model itself solves the above problem completely. It will appear that the Standard Model is a Complete Theory in itself and that it rules out any of the putative Unification Models like GUTs, Supersymmtry etc.

(II) Hiding the infinities

The physical mass m of the electron to order α is $m = m_0 + \delta m$ where m_0 is the bare mss of the electron. This is the mass of the electron in the absence of the electromagnetic interaction, that is mass of an uncharged electron [2]. This is an imporatnt point, that is to realize that the bare mass m_0 does not know of the electric charge. The idea of renormalizability is that m_0 which is physically not measurable (and may be infinite) is made to depend upon a cutoff Λ i.e. $m_0(\Lambda)$. We choose m_0 to depend upon Λ in such a manner that m is independent of Λ.

The same for the renormalized charge in QED [3] $e = (Z_3)^{\frac{1}{2}} e_0$ where e is the physically measured charge and e_0 is the bare charge of the electron. Note that this bare charge is independent of any mass parameter of the electron. Therefore the bare mass and the bare charge are independent of each other but of course, defining the same electron of which they are the primitive bare quantities.

QED is renormalizable in terms of two parameters, the bare mass m_0 and the bare charge e_0. When we calculate the true mass m and the true charge e (the actually measured quantities) we find that these expressions diverge. Next we regularize the theory by introducing an unphysical regulator Λ such that the resulting charge and mass are finite. That is

$$m = m(m_0, e_0, \Lambda)$$
$$e = e(m_0, e_0, \Lambda) \tag{1}$$

To make sense out of the regularizd theory we must somehow make m and e independent of Λ. Thus we make sure that m_0 and e_0 are functions of Λ such that m and e do not depend upon Λ. That is

$$\frac{dm}{d\Lambda} = 0 = \frac{\partial m}{\partial m_0}\frac{\partial m_0}{\partial \Lambda} + \frac{\partial m}{\partial e_0}\frac{\partial e_0}{\partial \Lambda} + \frac{\partial m}{\partial \Lambda}$$
$$\frac{de}{d\Lambda} = 0 = \frac{\partial e}{\partial m_0}\frac{\partial m_0}{\partial \Lambda} + \frac{\partial e}{\partial e_0}\frac{\partial e_0}{\partial \Lambda} + \frac{\partial e}{\partial \Lambda} \tag{2}$$

These coupled equations are first order and their initial conditions are provided by the experimentally measured or renormalized mass and charge. Thereon one uses the standard renormalization techniques [2]. The point that should be kept in mind is that the bare mass m_0 and the bare charge e_0 are two independent quantities.

Though playing a basic and fundamenatal role in cancelling the untraviolet divergences, to bring about finite and measaurable mass and charge, it is a puzzle as to wherefrom the bare mass and the bare charge arise. For mathematically oriented people, it may be a mere mathematical construct or a mathematical trick. While more physically minded persons, it may be a theoretical construct or as arising from some unkown "deeper theory" , that is deeper than QED [3]. Hence clearly its ontological status is quite fuzzy. In short, these bare quantities have a fundamental role in providing finite measuarable quantities in QED while their own existence and status is shrouded in mystery.

Note that when talking of the mathematical and physical consistency of QED there are really two properties to talk of. One is the bare charge and the bare mass, and next one is the correction terms like δm or Z_3 as above. When tackling the consistency problem of QED one normally just worries about the latter field theory correction terms. Conventionally one just ignores the bare mass and the bare charge terms, assuming these to arise mysteriously from somewhere. However these are not just simple physical parameters but are special in that these too can become infinite and also should spring from some deeper quantum field theory outside QED. It is not correct to ignore them. Afterall it is the interplay of infinities in these two properties that the finite physically measurable quantities arise. Hence one wishing to demonstrate the full consistency of QED cannot just confine oneself to only tacking the Landau ghost problem. It is only half the job done. One should also be able to solve the mystery of the bare quantities in QED as a renormalizable theory. As far as known to the author all the attempts as of now, involving demostartion of the mathematical consistency of the QED have only addressed the question of the Landau pole. These models have clearly failed to give the final word on the issue, mainly because they did not address the issue of the bare quantities in QED. We tackle both these issues in this paper.

(III) Mass and charge in the Standard Model

Let us define our model [4,5]. To start with let us first ignore the right-handed neutrino (note that we shall include it below) in say the first generation of particles. In the SM this is represented in the group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ as

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (3, 2, Y_q); u_R \sim (3, 1, Y_u); d_R \sim (3, 1, Y_d)$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \sim (1, 2, Y_l); e_R \sim (1, 1, Y_e) \tag{3}$$

Let us now define the electric charge in the most general way in terms of the diagonal generators of $SU(2)_L \otimes U(1)_Y$ as

$$Q = I_3 + bY \tag{4}$$

where b is an arbitrary parameter.

In the SM $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is spontaneously broken through a Higgs mechanism to the group $SU(3)_c \otimes U(1)_{em}$. Here the Higgs is assumed to be a doublet ϕ with arbitrary hypercharge Y_ϕ. The isospin $I_3 = -\frac{1}{2}$ component of the Higgs field develops a nonzero vacuum expectation value $< \phi >_o$. Since we want the $U(1)_{em}$ generator Q to be unbroken we require $Q < \phi >_o = 0$. This right away fixes b in (3) and we get

$$Q = I_3 + (\frac{1}{2Y_\phi})Y \tag{5}$$

To proceed further one imposes the anomaly cancellation conditions to establish constraints on the various hypercharges above. First $[SU(3)_c]^2 U(1)_Y$ gives $2Y_q = Y_u + Y_d$ and $[SU(2)_L]^2 U(1)_Y$ gives $3Y_q = -Y_l$. Next $[U(1)_Y]^3$ does not provide any new constraints. So the anomaly conditions themselves are not sufficient to provide quantization of electric charge in the SM. One has to provide new physical inputs to proceed further. There are two independent ways to do so.

Method I:

Here one demands that fermions acquire masses through Yukawa coupling in the SM. This brings about the following constraints:

$$Y_u = Y_q + Y_\phi; Y_d = Y_q - Y_\phi; Y_e = Y_l - Y_\phi \tag{6}$$

Note that $2Y_q = Y_u + Y_d$ from the anomaly cancellation condition for $[SU(3)_c]^2 U(1)_Y$ is automatically satisfied here from the Yukawa conditions above. Now using $3Y_q = -Y_l$ from anomaly cancellation, along with Yukawa terms above in $[U(1)_Y]^3$, does provide a new constrains of $Y_l = -Y_\phi$. Putting all these together one immediately gets charge quantization in the SM [4,5] as follows:

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, Y_q = \frac{Y_\phi}{3}; Q(u) = \frac{1}{2}(1 + \frac{1}{3}); Q(d) = \frac{1}{2}(-1 + \frac{1}{3})$$

$$u_R, Y_u = Y_\phi(1 + \frac{1}{3}); Q(u_R) = \frac{1}{2}(1 + \frac{1}{3}); d_R, Y_d = Y_\phi(-1 + \frac{1}{3}); Q(d_R) = \frac{1}{2}(-1 + \frac{1}{3})$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L; Y_l = -Y_\phi; Q(\nu) = 0, Q(e) = -1; e_R, Y_e = -2Y_\phi; Q(e_R) = -1 \tag{7}$$

Note that in the above quantization of the electric charge, Higgs hypercharge Y_ϕ always cancels out and hence remains unconstrained. A repetitive structure for the the other generations of particles gives charges for the other fermions as well.

Method II:

Next we ignore Yukawa coupling and impose the vector nature of the electric charge [4,5] which means that photon couples identically to the left handed and the right handed charges. That is $Q_L = Q_R$

$$\frac{1}{2}(1+\frac{Y_q}{Y_\phi}) = \frac{1}{2}\frac{Y_u}{Y_\phi}; giving : Y_u = Y_q + Y_\phi$$

$$Q(d) = \frac{1}{2}(-1+\frac{Y_q}{Y_\phi}) = \frac{1}{2}\frac{Y_d}{Y_\phi}; giving : Y_d = Y_q - Y_\phi$$

$$\frac{1}{2}(-1+\frac{Y_l}{Y_\phi}) = \frac{1}{2}\frac{Y_e}{Y_\phi}; giving : Y_e = Y_l - Y_\phi \tag{8}$$

And thereafter charge quantization as in method 1. Note that the hypercharge constraints arising by using the Yukawa terms as in Method I above in eqn. (6) are exacly the same as eqn. (8) by using Method II which guarantees the vector nature of the photon coupling. The singnificance of this equality/similarity shall be discussed below. Also note that the total charge is actually eQ (where after the spontaneous symmetry breaking e $= \frac{\sqrt{g_1 g_2}}{{g_1}^2+{g_2}^2}$).

Let us now add the right handed neutrino for the first generation irreducible representation given in eqn.(3). Let it be defined as (in the same notation):

$$\nu_R; (1,1,Y_\nu) \tag{9}$$

This brings in additional term from the Yukawa coupling given in eqn.(6) as

$$Y_\nu = Y_l + Y_\phi \tag{10}$$

Now $[U(1)_Y]^3$ anomaly condition, with all the Yukawa couplings and the $3Y_q = -Y_l$ condition, does not provide any new constraint on hypercharges. Only that it is consistent with the other conditions. Without ν_R it was this anomaly cancellation condition that gave crucial information which ensured charge quantization. Now with the incorporation of ν_R, the property of electric charge quantization is lost.

So we have charge quantization in the Standard Model and that is lost once we include the right-handed neutrino. Now we know that for any complete and closed mathematical structure which is found to provide a good description of nature, it is essential that some suitable boundary conditions be chosen. It is our experience, that without appropriate boundary conditions, a mathematical structure in itself does not provide enough predictive power to become useful in physics . As an example, we quote the Loretnz Group structure. The Special Theory of Relativity becomes well defined only with the boundary condition that the velocity of light be the limiting velocity.

In the same spirit we note that the Standard Model should include the right-handed component of neutrino also for a complete description of the particle reality. We do so. But for the model to become predictive we need atleast one boundary condition - and that is that we demand that the electric charge of the right handed neutrino be zero. Then we get all the predictive power of the SM as dicussed above. Just as without a limiting velocity, we cannt obtain the predictive power of the Lorentz transformation, similarly without a limiting charge of zero for the right-handed neutrino, we cannot obtain the predictive power of the quantized electric charge in the SM. Remember that a boundary condition cannot be derived from any fundmental theory. It is a physical reality which when judiciously imposed converts a mathematical structure into a possible physical reality.

If we so wish, we can look at this issue in another manner, which is more phenomenological in spirit. let us impose an empirical constraint. Let us assume that the the electric charge of this new entity - ν_R is zero. This would be consistent with the overall empirical reality, as any charged ν_R would have made its presence felt in the laboratory or in the cosmological data. Thus we are demanding it to be inert. Once we do that, we recover all the above results, as to charge quantizatiuon, without change.

So we have electric charge quantization in the Standard Model as shown above. But what does it mean that there are two independent methods - Method I and Metdod II to bringing about exactly the same kind of constraints on the hypercharges to finally provide charge quantization in the Standard Model. The significance of this fact is as follows.

Note that when Method I is adopted, Yukawa couplings define masses without knowing anything about the electric charge. So the masses obtained there are masses of an uncharged electron [2], exactly as what is needed for it to be the bare charge of the electron as an input in the renormalized charge which obviously arises once $U(1)_{em}$ has been brought into existence through $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ having been spontaneously broken through a Higgs mechanism to the group $SU(3)_c \otimes U(1)_{em}$. Hence the spontaneous symmetry breaking of $SU(2)_L \otimes U(1)_Y$ brings about Yukawa masses for the fermions which then act as bare masses as inputs for the newly generated $U(1)_{em}$ i.e. QED.

Since the early days of the Electroweak theory, it was always considered a weakness and a shortcoming of the Standard Model that it throws up so many unspecified mass parameters for all the fermions. As per our new iterpretation above, these unspecified mass paramerters in the Standard Model now become a VIRTUE of the Standard Moddel ! Clearly these unspecified bare masses take up any value, even infinite too, to counter the infinites in the δm term in the renormalized theory.

Next as per Method II above for charge quantization, shows that now the constraints that define charges are independnt of the Yukawa masses. So this charge does not know of the mass parameter. Hence these charges should be taken as bare charge inputs in the QED renomalization equations.

Thus we find that spontaneous breaking of $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ symmetry to $SU(3)_c \otimes U(1)_{em}$ provides the two independent inputs, the bare masses and the bare charges to allow for the calculation of renormalized masses and charges in $U(1)_{em}$ for QED. This consistency and self-completeness of the creation of bare masses and bare charges in QED is not only an indication that it is not only "not arbirary" but complete and consistent both physically and mathematically. It is also providing further strength to the Standard Model as being a Complete Theory in itself.

So the "deep theory" [3] that one needs to generate the bare mass and the bare charge for QED is none other than the poorly understood and the underappreciated, the Standard Model of particle physics. In fact as the Standard Model not only generates both the mass and the electric charge of the fermions consistently for the derived $U(1)_{em}$, it allows and thus ensures the renormalizability of the same quantities by providing them the necessary bare quantities. Thus QED needs the Standard Model to define fundamentally its structure.

Next to the issue of the Landau Pole (or Moscow Zero) in QED and which exists at scales much higher (at 10^{30} GeV) than the Planck Scale (at 10^{19}

GeV). None would disagree with the fact that it is spontaneous symmetry breaking of the electro-weak group that generates the masses. So as per the popular understanding, the matter particles are massless above the electro-weak scale and become massive after the symmetry breaking. But above we have also demonstrated very convincingly, that the electric charge also gets defined only after spontaneous symmetry breaking at the electro-weak scale. That is, above this scale, just as masses are not defined so also as well the charges are not defined. These two fundamental properties , the mass and the charge of the electron for example, get defined in the Standard Model only below the electro-weak symmetry breaking scale. Clearly the photon with its vector nature and couplings identically to the left-handed and the right-handed charges, also gets defined through the spontaneous symmetry breaking only. So clearly as per the Standard Model, there was no QED or the group $U(1)_{em}$ structure existng above the electro-weak breaking scale. This provides a natural cutoff for QED. Atually it is not just a cutoff. QED does not even exist above this scale. As QED does not even exist above the electro-weak breaking scale, the Landau pole is completely irrelevant scale for it. One would have taken Λ going from zero to infinity for QED in primitive times, when one did not know as to how QED actually got generated in the Standard Model. Earlier one believed that as a fundamental theory QED had universal existence at all energies and at all scales. But now we know that it is actually a derived theory coming into existence at a lower energy scale than visualizd before. Hence QED is both physically and mathematically a fully self-consistent theory in the energy range that it exists. Ofcourse one should not forget that this self-consistency also requires a consistent explanation of the bare quantities as done above.

(IV) About the various unification concepts

All the unification concepts which require that one may go beyond the Standard Model, in one way or the other, demand that there should be a group G such that

$$G \supset SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \tag{11}$$

where a larger group like SU(5) or SO(10) may encompass the Standard Model group. This is what one does in the Grand Unified Theories. Next due to some further reasons one introduces supersymmery in some manner.

The freedom to define the above group G and the freedom in introducing supersymmerty allows one to conceive of a large number of extensions beyond the Standard Model and which are currently focus of intense ongoing activities.

In all these theories it is assumed that the coupling constant of the Standard Model group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ runs. The scale at which the larger symmetry is attained, the unification is realized and fixed by the scale at which these three running coupling constants converge to. For GUTs it is roughly 10^{15} GeV, a few orders of magnitude smaller than the Planck scale. In all these models it is required that $U(1)_{em}$ exists generically right upto the unification scale and the three coupling consttant run continuously right upto there.

It is a common belief that the general idea of unification is on the "right track" [3, p. 421] by calculating the running of the coupling consstants as per different unification models. The best resulting fits [3, p. 271, 421, Fig 10] do not converge to a single point as per the SU)5) GUTs while for the Minimal Supersymmetric Model it does converge at approximately 10^{16} GeV. This was taken as a point in favour of MSSM.

But the Standard Model says something else. As we have shown above, $U(1)_{em}$ does not even exist above the electro-weak scale. This is in direct conflict with all the above ideas. As the Standard Model is solidly varified by the experiments in contrast to all the above extensions which as of now are empirically unconfirmed hypotheses. Hence the honour of determining what should be believed in, especially in a situation where there is a conflict between the two, should go to the Standard Model.

Gravity does not figure atall in the Standard Model. But we should not say that it is a weakness of the Stadnard Model that it fails to incorporate gravity in it. Actually this is a property of the Standard Model that describes the strong, the weak and'the electromagnetic interactions completely. Actually as a Complete Theory of these three interactions, it is not meant to describe gravity. For gravity we have to go outside the framework of the Standard Model. As'discussed above, as all the putative unification ideas beyond the Standard Model are wrong, we have to seek new grounds to be able to explain gravitational interaction as well. Very often books say that we actually ignore gravity in the Standard Model as it is so weak. But on the basis of what we have shown here - gravity is not part of the scheme of things in the Standard Model.

Hence we cannot accept any of these unification ideas and all these should be discarded, just as the false idea of the ether was discarded with the advent of the special theory of relativity at the beginning of the 20th century. Landau had tried to "bury with due honours" the quantum field theory of QED after dicovering the so called Landau singularity [6]. Now that we have shown that QED is a perfectly fine and a self-consistent quantum field theory, hence the same phrase may be used in a more appropriate present context, that the curently popular putative unification ideas like the GUTs etc should now be "buried with due honours".

References

1. R. P. Feynman, "QED: the strange theory of light and matter", Penguin, 1990

2. B. Hatfield, "Quantum Field Theory of point particles and strings", Addison-Wesley Pub. Co., New York, 1992

3. F. Wilczek, "Fantastic Realities", World Scientific Pub., Singapore, 2006

4. A Abbas, "Anomalies and charge quantization in the Standard Model with arbitrary number of colours", Phys. Lett B 238 (1990) 344

5. A Abbas, "Spontaneous symmetry breaking, quantization of the electric charge and the anomalies", J. Phys. G 16 (1990) L163

6. L. D. Landau, in "Niels Bohr and the development of physics", W. Pauli Ed., McGraw Hill, New York, 1955