Electric charge in the Standard Model of particle physics

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Abstract

At present the Standard Model is empirically the most successful model of particle physics. However, the electric charge which is a most basic and fundamental physical quantity, is popularly believed, not to be quantised in the Standard Model. This is what all the text books say. We look into the basis of the above statement and show that this arises from a gross misunderstanding of the mathematical structure of the Standard Model. We show that in contrast to the popularly held above view, the electric charge is consistently and fully quantized in the Standard Model. This arises from a consistent physical interpretation of the mathematical structure of the Standard Model. Lack of this understanding is being detrimental to further appreciation of the real significance of the Standard Model in the broad framework of particle physics.

Keywords: electric charge, Standard Model, spontaneus symmetry breaking, Higgs mechanism, anomalies

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(I) An historical introduction to electric charge in modern particle physics

Shortly after the discovery of neutron, Heisenberg in 1932 [1] put proton and neutron in together as a doublet of the group SU(2) (now called isospin group), to explain the charge independence of the strong interaction. Taking proton having eigenvalue T_3=1/2 and neutron as T_3 = -1/2 and given the fact that one can associate a baryon number B value of magnitude one, the electric charge for these can be defined as

$$Q = T_3 + \frac{B}{2} \tag{1}$$

This worked well for $(\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-})$ as T=3/2 quadruplet, (π^+, π^0, π^-) and (ρ^+, ρ^0, ρ^-) as T=1 triplet mesons and ω^0 as T=0 singlet meson.

The above expression for the electric charge worked for all the particles known until the late 40s, when strange particles started to make their presence felt [2]. When the Λ and K particles were discovered in the 1950s they were found to be produced copiusly but were observed to decay slowly with a long lifetime. It was postulated [2, p. 257] that they had a new quantum number, called Strangeness, associated with them, and which was conserved in the strong interaction to explain their copious production, but was violated in their weak decay. Let this new quantum number, Strangeness, be associated with a new U(1) group with one generator. The isospin group SU(2) above is now expanded to the group $SU(2) \otimes U(1)$ Therafter the Gell-Mann-Nishijima relation [3,4] was suggested for the electric charge of all the old and new particles in 1953 as

$$Q = T_3 + \frac{Y}{2} \tag{2}$$

where Y = B + S and where Y is called the Hypercharge as a generator of the U(1) group in the larger group $SU(2) \otimes U(1)$.

As we shall show below it was this expression for the $SU(2) \otimes U(1)$ of the strong interaction which was arbitrarily picked up by Glashow in 1961 for the weak interaction also. However in the meantime though (p,n) were providing the lowest dimensional fundamental representation of the isospin SU(2) group, it was $SU(2) \otimes U(1)$ of the strong interaction which was the basis of the Gell-Mann-Nishijima expression as given above. The straightfoward generalization of (p,n) the isospin group SU(2) to (p,n,Λ) for the group SU(3) within the so called Sakata model, failed to describe the baryons known at that time. Hence Gell-Mann [5] and Ne'eman [6] suggested late in 1961, that it was the octet of the mesons and the baryons, christened as the Eightfold-Way [7] which was what actually represented the larger group SU(3)s reality. Note that the fundamental representation of SU(3) played no role in the Eightfold-Way picture and it was the tensor reprentation that gave physical reality to the octet representation. That it was actually the three quarks that formed the three dimensional fundamental reprsentation came later in 1964 [8,9]. Gell-Mann however consisdered the Eightfold-Way to be the only physical reality while right until his Nobel Speech in 1969 he maiantained that the quarks were mere mathematical artefacts/fiction. He accepted the quarks as being physically real only after later complelling evidences from deep-inelastic scattering data from SLAC. So it was only later in 1964 that truly the Gell-Mann-Nishijima expression for the electric charge got associated with the larger group SU(3) of the strong interaction and the two terms got related the two diagonal generators of the group SU(3).

(II) Electric charge in the Standard Model

But in early 1961 it was the Gell-Mann-Nishijima expresion for the group $SU(2) \otimes U(1)$. And the Eightfold way was stll to come. It was that then Glashow while studying the weak interaction sought to incorporate electric charge in a larger electro-weak group as $SU(2)_W \otimes U(1)_W$ where the subscript W refers to the fact that one was not talking of the strong interaction (as what Gell-Mann and Nishijima were doing as above) but a different groups defining the Weak and Electromagnetic interactions in a partially unified manner. Glashow in early 1961 suggested that it is possible to take (ν, e) and (p,n) (note that neither the Higgs mechanism nor the quarks were known at that time) as left-handed and their corresponding right-handed partners (we shall discus these in more detail below) and he just picked up (actually at random) the Gell-mann-Nishijima definition of the electric chage for the weak group $SU(2)_W \otimes U(1)_W$ as

$$Q = {T_3}^W + \frac{Y_W}{2} \tag{3}$$

Here Y_W is called weak-hypercharge indicating its differetnt origin from the strong hypercharge above.

He then fixed the values of the weak-hypercharge Y_W to fit the various charges to the first generation of matter particles (ν, e) and (p,n). For example to get the correct electric charge for the left-handed electron and the corresponding ν he fixed $Y_W = -1$ and so on. Glashow was thus able to identify the group structure for the unified electro-weak theory in 1961. The use of the Higgs mechanism in this electroweak group came much later in 1964 through the work of Salam amd Weinberg.

Note that the above definition of the electric charge as defind by Glashow is arbitrary, mainly because of the fact that the weak-hypercharge defined as a generator of the group U(1) is not constrained to give any quantized value to Y_W. It can be any number and as indicatd above Glashow had to fix it by hand to give proper charges to the matter particles. Hence as per the above definition, electric charge cannot be said to be properly quantized in the Standard Model [11, p. 346]. Glashow arbitarily just picked up the Gell-Mann-Nishijima charge definition from strong interaction. His convention still is the most popular one today and a large number of text-books today are using the Glashow convention [ref 11, p. 346 and Ref 12 for a sample of books using Glashow convention] We may say about "half the particle physics world" uses the above convention to define electric charge is the Standard Model.

Note that because the above definition is just a convention, and also there is no reason why one should have the "ugly" factor of 1/2 attached to Y_W, and perhaps aesthetically it would be more satisfying to remove this factor of 1/2 and define the electric charge in the Electro-Weak group $SU(2)_W \otimes U(1)_W$ as per a new convention as

$$Q = {T_3}^W + Y_W \tag{4}$$

Hence the values of weak-hypercharges change with respect to the Glashow definition, e.g, for the (ν, e) now the value of Y_W = -1/2. Again all the values of hypercharges are fixed as per this new convention. We may say that the other "half of the particle physics world" prefers to use this other convention and we have listed several books doing so in Ref [13]. We have listed a large number of books and the reader is invited to pick up a book not listed here and find out which "half of the particle physics world" does it belong to.

If you do not want be part of the above two halves, you are welcome to join me in defining a new minority convention of electric charge as

$$Q = T_3{}^W + \frac{Y_W}{1947} \tag{5}$$

Now the value of the weak-hypercharge for the (ν, e) left handed doublet is Y_W = - 1947/2 and similarly for the other entities in the first generation of particles in the Standard Model.

So now we have defined three conventions - call the first one of Glashow as Convention I, the next one of Lee, Weinberg and others as Convention II and the last odd one as given above as Convention III. How is it possible? The reason is that first we are fixing the weak-hypercharges in each case to get the proper charges for the matter fields in each generation. The next important fact is that the anomalies in the Standard Models cancel generation by generation [11] and that these hypecharge assignments ensure that as they necessarily amount to Tr(Q) = 0 for each generation [11, p. 348], and thus these are always true for each definition separately.

However for a model as successful as the Standard Model of particle physics, it appears a a major weakness that electric charge is not quantized in it [11] and that the definition of the electric charge itself is conventional. As scientists, we would like our definitions of quantities, and that too as basic as the electric charge, not to be arbitray and conventional. So let us try to see, as to in what manner can we go beyond the limitations that we have imposed upon ourselves, as to the electric charge in the Standard Model.

(III) Going beyond conventions in the Standard Model

How do we go beyond the limits of the above conventions as to the electric charge in the Standard Model? A hint arises from the fact that all the three conventionlal definitions we had used only for the matter fields in each generation. Glashow had not known of the Higgs mechanism of sponatneous symmetry breaking in 1961. Let us now include and Higgs field also in the standard way as

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{6}$$

With an unknown weak-hypercharge defined as Y_ϕ. Now to get the correct charges for the Higgs field above we need

Convention I : $Y_\phi = 1$

Convention II: $Y_\phi = 1/2$

Convention III: $Y_\phi = 1947/2$

So with all these different definitions with fantastically different weak-hypecharges we still manage to get the correct electric charges for the Higgs fields. Since Higgs is the uniform and ubiquitous field providing all the well known properties to the Standard Model, is it possible that the above three results for the Higgs hypercharges are trying to tell us something.

A moments thought tells us that indeed it is so. All the three conventions basically for the the electric charge can be generalised to

$$Q = {T_3}^W + bY_W \tag{7}$$

where b is fixed so that the upper and lower components of the Higgs doublet get the correct charges. Immediately we see that it is when we take b $= \frac{1}{2Y_\phi}$. So the above electric charge becomes

$$Q = {T_3}^W + (\frac{1}{2Y_\phi})Y_W \tag{8}$$

Now we see that for $Y_W = Y_\phi$ the correct Higgs doublet charges arise. So clearly hidden within the above arbitaraineess of the three conventions, atleast for the Higgs doublet, is an exact definition of the electric charge which is not arbitrary or conventional. It is exact and gives correct quantized charges to the higgs doublet.

How about the matter fields? No problem, as long as we take their weak hypecharges to be proportinal to the Higgs Weak-hypercharge. For example for the first generation

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, Y_q = \frac{Y_\phi}{3}; Q(u) = \frac{1}{2}(1 + \frac{1}{3}); Q(d) = \frac{1}{2}(-1 + \frac{1}{3})$$

$$u_R, Y_u = Y_\phi(\frac{4}{3}); Q(u_R) = \frac{1}{2}(\frac{4}{3}); d_R, Y_d = Y_\phi(\frac{-2}{3}); Q(d_R) = \frac{1}{2}(\frac{-2}{3})$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L; Y_l = -Y_\phi; Q(\nu) = 0, Q(e) = -1; e_R, Y_e = -2Y_\phi; Q(e_R) = -1 \tag{9}$$

Thus only by including the Higgs doublet in a consistent manner and by looking deeply into the so called arbitrariness involed in the defintion of the electric charge in the Standard Model we find entirely on the basis of internal consistency and logic that the electric charge is unambigously defined in the Standard Model as $Q = T_3{}^W + (\frac{1}{2Y_\phi})Y_W$. This gives correct charges to the Higgs doublet and also with proper definitions of the weak-hypercharges for the matter field in each generation - which are always proportional to the Higgs weak-hypercharges, give the correct quantized electric charges in the Standard Model. Hence we state that contrary to the popular belief [11,12,13], the electric charge is consistently quantised in the Standard Model. Also we note that this quantization never fixes the Higgs weak-hypercharge.

So the arbitaraines in the definition of the electric charge, was trying to tell us somethig, in particluar about the Higgs doublet and also from there about the matter field charges. Once we had the correct definition of the electric charge in the Standard Model $Q = T_3{}^W + (\frac{1}{2Y_\phi})Y_W$, though we had to guess the other charges for the matter fields, the charges do came out exact and fully quantized. Hence our simple arguments show that electric charge is actully self-consistently quantized in the Stanadard Model and that it happens for each generation separately and that the Higgs weak-hypecharge never gets constrained to any value.

The above amazingly follows from simple consistency arguments within the Standard Model. Next, is it possible to obtain the above electric charge expressions in the Standard Model in a more mathematical and rigorous manner? Or is it possible that we may understand the various hypercharges defined as being proportional to the Higgs hypercharge and the correct definition of the electric charge in terms of the Higgs hypercharge as $Q = T_3{}^W + (\frac{1}{2Y_\phi})Y_W$ in a mathematically consistent manner? It is heartening to note that indeed it is so. Below we do that.

(IV) Mathematical charge quantization in the Standard Model

Let us start by looking at the first generation of quarks and leptons (u, d, e,ν) and assign them to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ representation as follows [14,15].

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, (3, 2, Y_q)$$

$$u_R; (3, 1, Y_u)$$

$$d_R; (3, 1, Y_d)$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}; (1, 2, Y_l)$$

$$e_R; (1, 1, Y_e) \tag{10}$$

To keep things as general as possible this brings in five unknown hypercharges.

Let us now define the electric charge in the most general way in terms of the diagonal generators of $SU(2)_L \otimes U(1)_Y$ as

$$Q' = a' I_3 + b' Y \tag{11}$$

We can always scale the electric charge once as $Q = \frac{Q'}{a'}$ and hence $(b = \frac{b'}{a'})$

$$Q = I_3 + bY \tag{12}$$

In the Standard Model $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is spontaniously broken through the Higgs mechanism to the group $SU(3)_c \otimes U(1)_{em}$. In this model the Higgs is assumed to be doublet ϕ with arbitrary hypercharge Y_ϕ. The isospin $I_3 = -\frac{1}{2}$ component of the Higgs develops a nonzero vacuum expectation value $< \phi >_o$. Since we want the $U(1)_{em}$ generator Q to be unbroken we require $Q < \phi >_o = 0$. This right away fixes b in (3) and we get

$$Q = I_3 + (\frac{1}{2Y_\phi})Y \tag{13}$$

Note that this is exactly the same as the eqn. (8) above. To proceed further one imposes the anomaly cancellation conditions to establish constraints on the various hypercharges above. First $[SU(3)_c]^2 U(1)_Y$ gives $2Y_q = Y_u + Y_d$ and $[SU(2)_L]^2 U(1)_Y$ gives $3Y_q = -Y_l$. Next $[U(1)_Y]^3$ does not provide any new constraints. So the anomaly conditions themselves are not sufficient to provide quantization of electric charge in the Standard Model. One has to provide new physical inputs to proceed further. Here one demands that fermions acquire masses through Yukawa coupling in the Standard Model. This brings about the following constraints:

$$Y_u = Y_q + Y_\phi; Y_d = Y_q - Y_\phi; Y_e = Y_l - Y_\phi \tag{14}$$

Note that $2Y_q = Y_u + Y_d$ from the anomaly cancellation condition for $[SU(3)_c]^2 U(1)_Y$ is automatically satisfied here from the Yukawa condition above. Now using $3Y_q = -Y_l$ from anomaly cancellation along with Yukawa terms above in $[U(1)_Y]^3$ does provide a new constrains of $Y_l = -Y_\phi$. Putting all these together one immediately gets charge quantization in the Standard Model [14,15] as follows:

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, Y_q = \frac{Y_\phi}{3},$$

$$Q(u) = \frac{2}{3}, Q(d) = \frac{-1}{3}$$

$$u_R, Y_u = \frac{3}{4}Y_\phi, Q(u_R) = \frac{2}{3}$$

$$d_R, Y_d = \frac{-2}{3}Y_\phi, Q(d_R) = \frac{-1}{3}$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}, Y_l = -Y_\phi, Q(\nu) = 0, Q(e) = -1$$

$$e_R, Y_e = -2Y_\phi, Q(e_R) = -1 \tag{15}$$

It has also been shown [14] that for arbitrary N_c the colour dependence of the electric charge as demanded by the Standard Model is

$$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c})$$

$$Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c}) \tag{16}$$

Note that within the Standard Model group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ the electroweak sector consists of $SU(2)_L \otimes U(1)_Y$. Still the electric charge knows of the colour degree of freedom. Interestingly though the electromagnetism does not know of the colour, still the electric charge has colour, existing within its guts, so to say. This shows that the Standard Model still is more unified than we had visualized so far.

Hence here we have shown that mathematically the structure of the Standard Model is such that the electric charge is fully quanitized in it. In adition, this structure, as obtained on the basis of our earlier phenomenological considerations, is fully supported by our mathemtical analysis here.

(V) Conclusions

We started with the standard textbook statements that in the Standard Model the definition of the electric charge is conventional and that it is not quantised. This is the understanding of the electric charge as given in all the text books. And this also is what is taught in all the graduate schools. Three conventions of defining the electric charge were discussed and thereafter we showed that incorporation of the Higgs field pointed to a hidden consistency in the electric charge in the Standard Model. That led us to the conclusion, on the basis of self-consistency arguments, that the electric charge is actually quantized in the Standard Model. Upto this point, this should be construed to be true as a phenomenologically consistent statement.

Next we showed that the same can now be put on a more solid mathematical basis. We have shown that the mathematical structure of the Standard Model is such that the electric charge if fully quantized in the Standard Model. Hence the phenomenological and the mathematical derivations give identical results. Hence not knowing this basic fact will be detrimental to a proper understanding of particle physics in the future. Therefore it is pointed out that all the text books should be corrected to account for this new basic reality of the Standard Model.

Note that in the above quantization of the electric charge, the Higgs hypercharge Y_ϕ always cancels out and hence remains unconstrained. A few others, while looking at the electric charge in the Standard Model, had ignord this fact as indicated in Ref [14,15,16]. This is important as not paying attention to this fact can lead to asking wrong questions, as for example for the possibility of milli-charged particles, which are ruled out by the Standard Model as per the above arguments [15, 16]. Note that a repetitive structure gives charges for the other generation of fermions as well. Hence the electric charge quantization as indicated here and as discussed in ref [14,15,16] is based on physically rigorous and mathematically consistent arguments.

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