SKIN EFFECT PROBLEM WITH DIFFUSION BOUNDARY CONDITIONS IN MAXWELL PLASMA

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Abstract

The analytical method of solving the boundary problems for a system of equations describing the behaviour of electrons and an electric field in the Maxwell plasma half-space is developed. Here the diffusion reflection of electrons from the surface as a boundary condition is supposed. The exact solution for boundary value problem of the Vlasov – Maxwell equations through the eigenfunctions of the discrete spectrum and the continuous spectrum is obtained. The method of decomposition by eigenfunctions of discrete and continuous spectrum is used. The exact expression for the surface impedance also is obtained.

1. Introduction. Statement of problem. The skin effect is the plasma's response to an external constant-amplitude variable electromagnetic field that is tangential to the surface [1]. This problem represents essential interest [1]–[5], with the focal point being on the computation of surface impedance.

The problem about skin effect is actual and is regularly studied and last years (see, for exampe, [5]).

The paper further develops the analytical method of solving boundary problem for a system of equations describing behaviour of electrons and electric field in Maxwell plasma. The method is based on the idea of the decomposition of the solution the problem by singular eigenfunctions of the corresponding characteristic equations.

We find the analytical solution to the boundary problem of the skin effect theory for electron plasma that fills the half-space. The analytical solution has a form of the sum of an integral and two (or one) exponentially decreasing particular solutions of the initial system. Depending on index the problem one of the particular solutions disappears.

We consider a collision plasma with behaviour is described by modelling kinetic equation with integral of collisions in the form BGK–model (BGK=Bhatnagar, Gross, Krook) (see, for example, [4]).

Let's Maxwell plasma fills the half-space $x > 0$. Here x is the orthogonal coordinate to the plasma boundary. Let's the external electric field has only y component. Then the self-consistent electric field inside in plasma also has only y component $E_y(x,t) = E(x)e^{-i\omega t}$. We consider the kinetic equation for the electron distribution function:

$$\frac{\partial f}{\partial t} + \mathrm{v}_x \frac{\partial f}{\partial x} + eE(x)e^{-i\omega t}\frac{\partial f}{\partial p_y} = \nu(f_0 - f(t,x,\mathbf{v})). \tag{1.1}$$

In equation (1.1) ν is the frequency of electron collisions with ions, e is the charge of electron, $f_0(\mathrm{v})$ is the equilibrium Maxwell distribution function, $\mathbf{p}$ is the momentum of electron,

$$f_0(\mathrm{v}) = n\left(\frac{\beta}{\pi}\right)^{3/2}\exp(-\beta^2\mathrm{v}^2), \quad \beta = \frac{m}{2k_BT}.$$

Here m is the mass of electron, k_B is the Boltzmann constant, T is the temperature of plasma, v is the velocity of the electron, n is the concentration of electrons, c is the speed of light.

The electric field $E(x)$ satisfies to the equation:

$$E''(x) + \frac{\omega^2}{c^2}E(x) = -\frac{4\pi i e^{i\omega t}\omega e}{c^2}\int v_y f(t,x,\mathbf{v})\,d^3v. \tag{1.2}$$

We assume that intensity of an electric field is such that linear approximation is valid. Then distribution function can be presented in the form:

$$f = f_0\left(1 + C_y\exp(-i\omega t)h(x,\mu)\right),$$

where $\mathbf{C} = \sqrt{\beta}\mathrm{v}$ is the dimensionless velocity of electron, $\mu = C_x$. Let $l = v_T\tau$ is the mean free path of electrons, $v_T = 1/\sqrt{\beta}$, v_T is the thermal electron velocity, $\tau = 1/\nu$. We introduce the dimensionless parameters and the electric field:

$$t_1 = \nu t, \quad x_1 = \frac{x}{l}, \quad e(x_1) = \frac{\sqrt{2}e}{\nu\sqrt{mk_BT}}E(x_1).$$

Later we substitute x_1 for x. The substitution produces the following form of the kinetic equation (1.1) and the equation on a field with the displacement current (1.2):

$$\mu\frac{\partial h}{\partial x} + z_0\,h(x,\mu) = e(x), \quad z_0 = 1 - i\omega\tau, \tag{1.3}$$

$$e''(x) + Q^2e(x) = -i\frac{\alpha}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\exp(-\mu'^2)\,h(x,\mu')\,d\mu', \quad Q = \frac{\omega l}{c}, \tag{1.4}$$

where $\delta=\dfrac{c^2}{2\pi\omega\sigma_0}$, δ is the classical depth of the skin layer, $\sigma_0=\dfrac{e^2n}{m\nu}$, σ_0 is the electric conductance, $\alpha=\dfrac{2l^2}{\delta^2}$, α is the anomaly parameter.

We formulate the boundary conditions for the distribution function of the electron in case of the diffusion electron reflection from the surface:

$$h(0,\mu)=0, \qquad 0<\mu<+\infty. \tag{1.5}$$

We use the condition that function $h(x,\mu)$ vanishes far from the surface:

$$h(+\infty,\mu)=0, \qquad -\infty<\mu<+\infty, \tag{1.6}$$

and conditions for electric field on the interface and far from it:

$$e'(0)={e_s}', \qquad e(+\infty)=0, \tag{1.7}$$

where ${e_s}'$ is the given value of the gradient of the electric field on the plasma interface.

So, the skin effect problem is formulated completely. We seek solution of system of the equations (1.3) and (1.4) in this problem, that satisfy to the boundary conditions (1.5) – (1.7).

2. Eigenfunctions and eigenvalues. The separation of variables [2]–[4]

$$h_\eta(x,\mu)=\exp(-z_0\frac{x}{\eta})\Phi(\eta,\mu), \tag{2.1}$$

$$e_\eta(x)=\exp(-z_0\frac{x}{\eta})E(\eta), \tag{2.2}$$

where η is the spectral parameter (generally, complex-valued), reduces system of the equation (1.3) and (1.4) to the characteristic system:

$$(\eta-\mu)\Phi(\eta,\mu)=\frac{\eta}{z_0}E(\eta), \tag{2.3}$$

$$\left[z_0^2+\left(\frac{\omega l}{c}\right)^2\eta^2\right]E(\eta)=-\frac{i\alpha\eta^2}{\sqrt{\pi}}n(\eta), \tag{2.4}$$

where

$$n(\eta)=\int\limits_{-\infty}^{\infty}e^{-\mu^2}\Phi(\eta,\mu)d\mu.$$

From equations (2.3) and (2.4) for $\eta\in(-\infty,\infty)$, we find the eigenfunctions of the continuous spectrum [2], [6]:

$$\Phi(\eta,\mu)=\frac{a}{\sqrt{\pi}}\eta^3e^{-\eta^2}P\frac{1}{\eta-\mu}+\lambda(\mu)\delta(\eta-\mu), \tag{2.6}$$

$$E(\eta) = \frac{az_0}{\sqrt{\pi}}\eta^2 e^{-\eta^2}, \quad a = -i\frac{\alpha}{z_0^3}. \tag{2.7}$$

In the equation (2.6) the symbol Px^{-1} denotes the distribution, i.e. the principal value of the integral of x^{-1}, $\delta(x)$ is the Dirac delta function.

Since the distribution and electric field functions decline as we go further from the boundary, we regard the continuous as positive real half-axis: $0 < \eta < +\infty$. The eigen solution of the continuous spectrum $h_\eta(x,\mu)$, $e_\eta(x)$ are decreasing from value x, since $\operatorname{Re} z_0 > 0$.

In the equations (2.6) and (2.7) there is the normalization condition:

$$\int\limits_{-\infty}^{\infty} e^{-\mu^2}\Phi(\eta,\mu)d\mu = \left[1 + \left(\frac{\omega l}{c}\right)^2 \eta^2\right] e^{-\eta^2},$$

$\lambda(z)$ is the dispersion function,

$$\lambda(z) = 1 + \left(\frac{\omega l}{cz_0}\right)^2 z^2 + \frac{az^3}{\sqrt{\pi}} \int\limits_{-\infty}^{\infty} \frac{e^{-\mu^2}d\mu}{\mu - z}. \tag{2.8}$$

Using the method developed in [2, 4], we determine the structure of the discrete spectrum of the characteristic system equations (2.3) – (2.4). By definition, this spectrum consists of the zeros of the dispersion function that do not belong to the cut $(-\infty,\infty)$.

Let N is the number of such zeros. Taking into account the fact that the dispersion function has a double pole at $z=\infty$, the number of its zeros is

$$N = 2 + \frac{1}{2\pi}[\arg\lambda(z)]_{\gamma_\varepsilon}, \tag{2.9}$$

where γ_ε is the contour that encompasses the cut that is at the distance ε from the former cut $(-\infty,\infty)$ and does not contain zeros of the dispersion function.

Passing to the limit in the equation (2.9) as $\varepsilon \to 0$, we obtain

$$N = 2 + \frac{1}{2\pi}\left[\arg\frac{\lambda^+(\tau)}{\lambda^-(\tau)}\right]_{(-\infty,\infty)} = 2 + \frac{1}{\pi}\left[\arg\frac{\lambda^+(\tau)}{\lambda^-(\tau)}\right]_{(0,\infty)}.$$

Here $\lambda^\pm(\tau)$ are the boundary values of the function $\lambda(z)$ above and below the cut $(-\infty,\infty)$,

$$\lambda^\pm(\mu) = \lambda(\mu) \pm i\pi a\mu^3\exp(-\mu^2),$$

where $\lambda(\mu)$ it is defined by (2.8), and the integral here is understood in sense of a principal value (according Cauchy).

The fact that $\lambda(\mu)$ has a double pole at $z=\infty$ follows from the asymptotic expansion in the Laurent series:

$$\lambda(z)=1+az^2\sqrt{\pi}+a\frac{\sqrt{\pi}}{2}+a\int\limits_{-\infty}^{\infty}\frac{\tau^3\exp(-\tau^2)}{\tau-z}d\tau,\quad z\to\infty.$$

We consider the domain D^+ (we denote its exterior by D^-) on the Λ-plane (fig. 1) with the boundary given by the parametric equations:

$$\partial D^+=\{\delta=\delta_1+i\delta_2:\mathrm{Re}\ \lambda^+(\mu)=0,\mathrm{Im}\ \lambda^-(\mu)=0,\quad -\infty\leqslant\mu\leqslant+\infty\}.$$

Here δ is a new parameter,

$$\delta=\delta_1+i\delta_2=-\frac{4c^2}{\pi v_T^2\omega_p^2}\cdot\frac{(\omega+i\nu)^2}{\omega},$$

$$\delta_1=-\frac{\omega^3-3\omega\nu^2}{v_c^2\omega\omega_p^2},\quad \delta_2=-\frac{3\omega^2\nu-\nu^3}{v_c^2\omega\omega_p^2},\quad v_c=\sqrt{\frac{\pi}{4}\frac{v_T}{c}}.$$

Besides, the boundary of the domain D^+ can be written in the explicit form:

$$\partial D^+=\{\delta=\delta_1+i\delta_2:\delta_1=p(\mu),\delta_2=q(\mu),\quad -\infty\leqslant\mu\leqslant+\infty\},$$

where

$$p(z)=-\frac{z^3}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\frac{\exp(-\mu^2)\,d\mu}{\mu-z},\qquad q(z)=\sqrt{\pi}z^3\exp(-z^2).$$

As in [4], it can be shown that:

1) if $\delta\in D^+$, then $N=4$ and the problem index $\varkappa(G)=1$;

2) if $\delta\in D^-$, then $N=2$ and $\varkappa(G)=0$.

Here $\varkappa=\varkappa(G)$ is the index of the problem, $G(\mu)=\dfrac{\lambda^+(\mu)}{\lambda^-(\mu)}$,

$$\varkappa(G)=\mathrm{ind}_{[-\infty,\infty]}G(\mu)=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}d\arg\frac{\lambda^+(\mu)}{\lambda^-(\mu)}.$$

The boundary mode, i.e., the case $\delta\in\partial D$, is not considered in this paper, since it was analyzed in details in [2].

We find the general solution to system (1.3), (1.4) in the form of an expansion in the eigenfunctions of the discrete and continuous spectrum; this solution automatically satisfies the boundary conditions at infinity:

$$h(x,\mu)=\frac{a}{\sqrt{\pi}}\sum_{k=0}^{1}\frac{A_k\eta_k^3}{\eta_k-\mu}\exp\Big(-\eta_k^2-\frac{z_0x}{\eta_k}\Big)+$$

$$+\int_0^\infty \exp\Big(-\frac{z_0x}{\eta}\Big)A(\eta)\Phi(\eta,\mu)\,d\eta, \tag{2.10}$$

$$e(x)=\frac{az_0}{\sqrt{\pi}}\sum_{k=0}^{1}A_k\eta_k^2\exp\Big(-\eta_k^2-\frac{z_0x}{\eta_k}\Big)+$$

$$+\frac{az_0}{\sqrt{\pi}}\int_0^\infty \exp\Big(-\eta^2-\frac{z_0x}{\eta}\Big)\eta^2\,A(\eta)\,d\eta. \tag{2.11}$$

Here $A_k (k=0,1)$ are the unknown coefficients corresponding to the discrete spectrum, $A_1=0$, if $\delta\in D^-$; and $A(\eta)$ is an unknown function called the coefficient of the continuous spectrum $\operatorname{Re}(z/\eta_k)>0 \quad (k=0,1)$, $\operatorname{Re}\, z_0=1$.

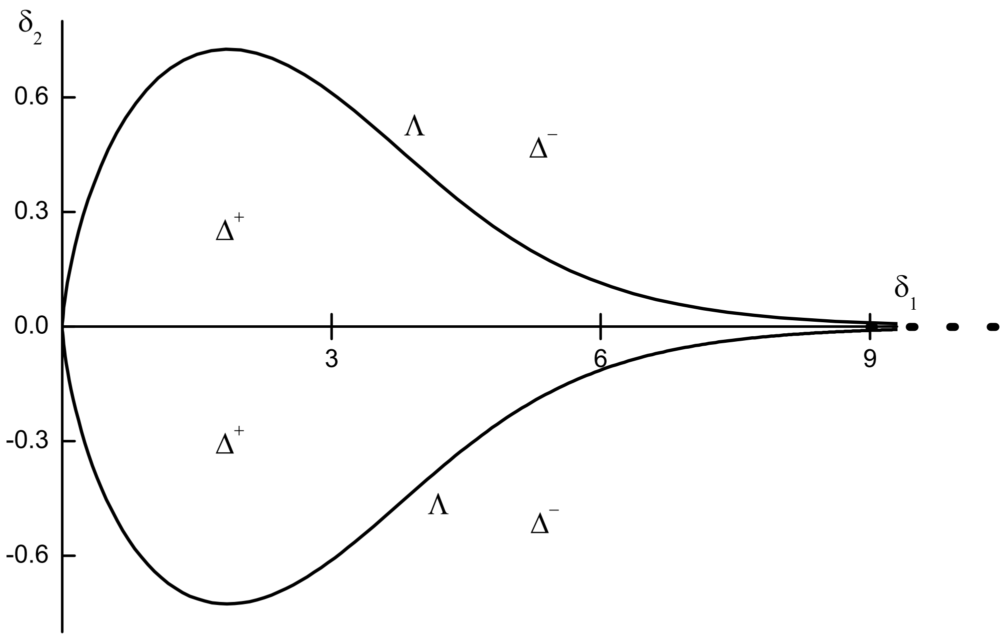

Fig. 1.

3. The index of the problem equals zero. We consider the case when the problem index is equal to zero. Substituting expressions (2.10) and (2.11) into (1.5), we obtain the singular integral equation with the Cauchy kernel:

$$\frac{a}{\sqrt{\pi}}\int_0^\infty \frac{\eta^3\exp(-\eta^2)A(\eta)}{\eta-\mu}\,d\eta+\lambda(\mu)A(\mu)+a\varphi(\mu)=0,\quad 0<\mu<\infty, \tag{3.1}$$

where

$$\varphi(\mu)=\frac{A_0}{\sqrt{\pi}}\frac{\eta_0^3\exp(-\eta_0^2)}{\eta_0-\mu}.$$

We define the auxiliary function in the complex plane

$$N(z)=\frac{1}{\sqrt{\pi}}\int\limits_0^\infty\frac{\eta^2\exp(-\eta^2)A(\eta)}{\eta-z}\,d\eta.$$

Then the equation (3.1) is possible to transform to the Riemann — Hilbert boundary value problem:

$$\lambda^+(\mu)\Big[N^+(\mu)+\varphi(\mu)\Big]=\lambda^-(\mu)\Big[N^-(\mu)+\varphi(\mu)\Big],\quad 0<\mu<\infty. \tag{3.2}$$

We consider the Riemann — Hilbert boundary value problem on the half-axis:

$$\frac{X^+(\mu)}{X^-(\mu)}=G(\mu),\qquad 0<\mu<+\infty.$$

where

$$G(\mu)=\frac{\lambda^+(\mu)}{\lambda^-(\mu)},$$

$G(\mu)$ is the coefficient of the boundary value problem.

The index in this problem is $\varkappa(G)=0$, then the solution of problem (3.2) has the form [4]:

$$X(z)=\exp V(z),\qquad V(z)=\frac{1}{2\pi i}\int\limits_0^\infty\frac{\ln G(\tau)\,d\tau}{\tau-z}. \tag{3.3}$$

We use (3.3) to reduce problem (3.2) for determining an analytic function $N(z)$ from its zero jump problem on the cut:

$$X^+(\mu)\Big[N^+(\mu)+\varphi(\mu)\Big]=X^-(\mu)\Big[N^-(\mu)+\varphi(\mu)\Big],\quad 0<\mu<\infty. \tag{3.4}$$

A general solution of (3.4) is given by the formula

$$N(z)=\frac{A_0}{\sqrt{\pi}}\frac{\eta_0^3\exp(-\eta_0^2)}{z-\eta_0}+\frac{C_0}{(z-\eta_0)X(z)}. \tag{3.5}$$

where C_0 is an arbitrary constant.

Let's eliminate poles at the decision (3.5), we have:

$$C_0=-\frac{A_0}{\sqrt{\pi}}\exp(-\eta_0^2)\eta_0^3X(\eta_0). \tag{3.6}$$

Employing the Sokhotsky formula for difference from the boundary values of function $N(z)$, we find the coefficient of the continuous spectrum:

$$2\sqrt{\pi}\, i\, \eta^3 \exp(-\eta^2)\, A(\eta) = \frac{C_0}{\eta-\eta_0}\Big[\frac{1}{X^+(\eta)}-\frac{1}{X^-(\eta)}\Big].$$

From the definition of the auxiliary function $N(z)$ we have:

$$N(0) = \frac{1}{\sqrt{\pi}}\int\limits_0^\infty \eta^2 \exp(-\eta^2) A(\eta) d\eta. \tag{3.7}$$

Considering equality (3.7), we write (2.11) in form:

$$-\frac{C_0}{\eta_0 X(\eta_0)} + N(0) = \frac{1}{az_0}.$$

From general solution (3.5) the constant C_0 is found:

$$C_0 = -\frac{\eta_0}{az_0} X(0). \tag{3.8}$$

From comparison (3.6) and (3.8) we get the unknown coefficients corresponding to the discrete spectrum:

$$A_0 = \frac{\sqrt{\pi}\, X(0)}{a\, z_0\, X(\eta_0)\, \eta_0^2\, \exp(-\eta_0^2)}.$$

The derivative of expression (2.11) leads us to equality

$$\frac{e'(x)}{az_0^2} = \frac{C_0}{\eta_0^2 X(\eta_0)} \exp(-\frac{z_0 x}{\eta_0}) - \frac{1}{\sqrt{\pi}}\int\limits_0^\infty \exp(-\eta^2 - \frac{z_0 x}{\eta})\eta A(\eta) d\eta.$$

From here we find the electric field derivative on plasma boundary $x = 0$:

$$\frac{e'(0)}{az_0^2} = \frac{C_0}{\eta_0 X(\eta_0)} - \frac{1}{\sqrt{\pi}}\int\limits_0^\infty \exp(-\eta^2)\eta A(\eta) d\eta. \tag{3.9}$$

For calculation of integral from the expression (3.9) we shall use a derivative of auxiliary function

$$N'(\mu) = \frac{1}{\sqrt{\pi}}\int\limits_0^\infty \frac{\eta^3 \exp(-\eta^2) A(\eta)\, d\eta}{(\eta-\mu)^2}. \tag{3.10}$$

Using the expression (3.10) we find the derivative in zero:

$$N'(0)=\frac{1}{\sqrt{\pi}}\int\limits_0^\infty \eta\exp(-\eta^2)A(\eta)\,d\eta. \tag{3.11}$$

Using expression for coefficient of the continuous spectrum by (3.11) we find:

$$N'(z)=-\frac{C_0}{(z-\eta_0)^2}\Big(\frac{1}{X(z)}-\frac{1}{X(\eta_0)}\Big)-\frac{C_0}{z-\eta_0}\frac{X'(z)}{X^2(z)},$$

whence

$$N'(0)=-\frac{C_0}{\eta_0^2}\Big(\frac{1}{X(0)}-\frac{1}{X(\eta_0)}\Big)-\frac{C_0}{\eta_0}\frac{X'(0)}{X^2(0)}. \tag{3.12}$$

Substituting (3.12) into (3.9) we obtain the equation:

$$e'(0)=\frac{az_0^2C_0}{\eta_0X(0)}\left[\frac{1}{\eta_0}-\frac{X'(0)}{X(0)}\right]=z_0\left[\frac{X'(0)}{X(0)}-\frac{1}{\eta_0}\right]. \tag{3.13}$$

In [2] the following formula for the impedance is given:

$$Z=\frac{4\pi i\omega l}{c^2z_0}\cdot\frac{e(0)}{e'(0)}. \tag{3.14}$$

Substituting (3.13) into (3.14), we obtain the exact expression for the impedance:

$$Z=\frac{4\pi i\omega l}{c^2\,z_0}\left[\frac{X'(0)}{X(0)}-\frac{1}{\eta_0}\right]^{-1}. \tag{3.15}$$

The equation (3.15) is the exact expression for calculation of the surface impedance. This equation expresses value of the impedance in terms of the function $X(z)$ and zeros of the dispersion function of the problem.

4. The index of the problem equals one. Now we consider case when the problem index is equal one, i.e. $\varkappa(G)=1$.

Substituting expressions (2.10) and (2.11) into (1.5), we obtain the singular integral equation with the Cauchy kernel:

$$a\varphi(\mu)+\frac{a}{\sqrt{\pi}}\int\limits_0^\infty\frac{\eta^3\exp(-\eta^2)A(\eta)}{\eta-\mu}\,d\eta+\lambda(\mu)A(\mu)=0,\quad 0<\mu<\infty, \tag{4.1}$$

where

$$\varphi(\mu)=\frac{1}{\sqrt{\pi}}\sum_{k=0}^{1}\frac{A_k\eta_k^3\exp(-\eta_k^2)}{\eta_k-\mu}.$$

We define the auxiliary function in the complex plane:

$$N(z)=\frac{1}{\sqrt{\pi}}\int\limits_0^\infty\frac{\eta^3\exp(-\eta^2)A(\eta)}{\eta-z}\,d\eta,$$

for which boundary values from above and from below on the valid axis formulas of Sokhotskii are carried out:

$$N^+(\mu)-N^-(\mu)=2\sqrt{\pi}\,i\,\mu^3\exp(-\mu^2)A(\mu).$$

Using the boundary values $N^{\pm}(\mu)$ and $\lambda^{\pm}(\mu)$, we pass from the singular equation (4.1) to the Riemann — Hilbert boundary value problem:

$$\lambda^+(\mu)[N^+(\mu)+\varphi(\mu)]=\lambda^-(\mu)[N^-(\mu)+\varphi(\mu)],\quad 0<\mu<\infty. \qquad (4.2)$$

Let's solve a corresponding Riemann — Hilbert boundary value problem:

$$\frac{X^+(\mu)}{X^-(\mu)}=G(\mu),\qquad 0<\mu<+\infty, \qquad (4.3)$$

where

$$G(\mu)=\frac{\lambda^+(\mu)}{\lambda^-(\mu)},\qquad 0<\mu<\infty.$$

Since problem index is equal one $\varkappa(G)=1$ as the solution of (4.3) we take function

$$X(z)=\frac{1}{z}\exp V(z),\quad V(z)=\frac{1}{2\pi i}\int\limits_0^\infty\frac{\ln G(\tau)-2\pi i}{\tau-z}\,d\tau. \qquad (4.4)$$

Using (4.4) we transform the boundary value problem (4.2) to determining an analytic function from its zero jump problem on the cut:

$$X^+(\mu)[N^+(\mu)+\varphi(\mu)]=X^-(\mu)[N^-(\mu)+\varphi(\mu)],\quad 0<\mu<\infty. \qquad (4.5)$$

The general solution of (4.5) is following

$$N(z)=\frac{1}{\sqrt{\pi}}\sum_{k=0}^{1}\frac{A_k\eta_k^3\exp(-\eta_k^2)}{z-\eta_k}+\sum_{k=0}^{1}\frac{C_k}{(z-\eta_k)X(z)}, \qquad (4.6)$$

where C_0, C_1 is the arbitrary values.

Since $N(\infty)=0$, from here we receive that $C_0+C_1=0$.

Let's eliminate poles at the decision (4.6), we get:

$$C_0=-\frac{A_0}{\sqrt{\pi}}\eta_0^3\exp(-\eta_0^2)X(\eta_0),\quad C_1=-\frac{A_1}{\sqrt{\pi}}\eta_1^3\exp(-\eta_1^2)X(\eta_1). \qquad (4.7)$$

Substituting the general solution (4.6) in the Sokhotskii formula for (4.6) we find unknown function called the coefficient of the continuous spectrum:

$$\eta^3\exp(-\eta^2)A(\eta)=\frac{1}{2\sqrt{\pi}\,i}\left[\frac{1}{X^+(\eta)}-\frac{1}{X^-(\eta)}\right]\sum_{k=0}^{1}\frac{C_k}{\eta-\eta_k}. \tag{4.8}$$

Using (4.7) we reduce (2.10) to the form:

$$-\frac{C_0}{\eta_0X(\eta_0)}-\frac{C_1}{\eta_1X(\eta_1)}+\frac{1}{\sqrt{\pi}}\int\limits_0^\infty\eta^2\exp(-\eta^2)A(\eta)d\eta=\frac{1}{az_0}, \tag{4.9}$$

From equation (4.9) we will find last integral using definition of function $N(z)$:

$$N(0)=\frac{1}{\sqrt{\pi}}\int\limits_0^\infty\eta^2\exp(-\eta^2)A(\eta)d\eta.$$

Using the common decision (4.6), we found $N(0)$:

$$N(0)=C_0\left[\frac{1}{\eta_0X(\eta_0)}-\frac{1}{\eta_1X(\eta_1)}-\frac{1}{\eta_0X(0)}+\frac{1}{\eta_1X(0)}\right]$$

We substitute this expression into the equation (4.9):

$$C_0=\frac{X(0)}{az_0\Big(\dfrac{1}{\eta_1}-\dfrac{1}{\eta_0}\Big)}. \tag{4.10}$$

Differentiating expansion for electric field (2.11) and using (4.7), where $x=0$ we found:

$$e'(0)=\frac{C_0az_0^2}{\eta_0^2X(\eta_0)}-\frac{C_0az_0^2}{\eta_1^2X(\eta_1)}-\frac{az_0^2}{\sqrt{\pi}}\int\limits_0^\infty\eta\exp(-\eta^2)A(\eta)\,d\eta. \tag{4.11}$$

Integral from (4.11) is the auxiliary function derivative $N'(0)$:

$$N'(0)=\frac{1}{\sqrt{\pi}}\int\limits_0^\infty\eta\exp(-\eta^2)A(\eta)\,d\eta.$$

The common solution (4.6) we present in form:

$$N(z)=\frac{1}{z-\eta_0}\left[\frac{C_0}{X(z)}+\frac{1}{\sqrt{\pi}}A_0\eta_0^3\exp(-\eta_0^2)\right]+$$

$$+\frac{1}{z-\eta_1}\Bigg[\frac{C_1}{X(z)}+\frac{1}{\sqrt{\pi}}A_1\eta_1^3\exp(-\eta_1^2)\Bigg].$$

Derivative of this expression is equal:

$$N'(z)=-\frac{1}{(z-\eta_0)^2}\Bigg[\frac{1}{X(z)}-\frac{1}{X(\eta_0)}\Bigg]-\frac{C_0}{z-\eta_0}\frac{X'(z)}{X^2(z)}+$$

$$+\frac{C_0}{(z-\eta_1)^2}\Bigg[\frac{1}{X(z)}-\frac{1}{X(\eta_1)}\Bigg]+\frac{C_0}{z-\eta_1}\frac{X'(z)}{X^2(z)}.$$

Hence, we obtain:

$$N'(0)=-\frac{1}{\eta_0^2}\Bigg[\frac{1}{X(0)}-\frac{1}{X(\eta_0)}\Bigg]+\frac{C_0}{\eta_0}\frac{X'(0)}{X^2(0)}+$$

$$+\frac{C_0}{\eta_1^2}\Bigg[\frac{1}{X(0)}-\frac{1}{X(\eta_1)}\Bigg]-\frac{C_0}{\eta_1}\frac{X'(0)}{X^2(0)}.$$

Using this equality for the electric field derivative we have:

$$e'(0)=\frac{C_0az_0^2}{X(0)}\Bigg[\Big(\frac{1}{\eta_0}-\frac{1}{\eta_1}\Big)\Big(\frac{1}{\eta_0}+\frac{1}{\eta_1}\Big)+\frac{X'(0)}{X(0)}\Big(\frac{1}{\eta_0}-\frac{1}{\eta_1}\Big)\Bigg].$$

Therefore according to (4.10) we receive that

$$e'(0)=z_0\Bigg[\frac{X'(0)}{X(0)}-\frac{1}{\eta_0}-\frac{1}{\eta_1}\Bigg].$$

Hence, expression for the impedance is equal:

$$Z=\frac{4\pi i\omega l}{c^2z_0}\cdot\Bigg[\frac{X'(0)}{X(0)}-\frac{1}{\eta_0}-\frac{1}{\eta_1}\Bigg]^{-1}.$$

This equation expresses value of the impedance in terms of factor function $X(z)$ and zeros of dispersive function of the problem.

CONCLUSIONS

A closed form solution of the system of two equations Boltzmann — Vlasov and Maxwell arising in the skin effect problem for a Maxwellian plasmas is presented. The kinetic Boltzmann — Vlasov equation with a τ–model collision operator is considered. Case's [7] method and Riemann — Hilbert boundary value problem [8] with the coefficient $G(\mu)=\dfrac{\lambda^+(\mu)}{\lambda^-(\mu)}$ is utilized, here $\lambda(z)$ is the dispersion function of the problem. The exact solution of the initial boundary value problem with diffusion scattering of electrons from plasma boundary is constructed. The exact formula for calculation of the surface impedance is obtained.

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