Scattering by a slab: an exact calculation

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Abstract

The model consists of an infinite number of point interactions distributed periodically on a finite number of parallel planes. Exact formulas for the transmission coefficients are given. Numerical behavior is presented, first in the case the incident plane wave has a wave vector perpendicular to the planes and then for some tilted directions. For some values of the interaction parameter the reflection coefficient is close to one for a relatively large set of incident wavelengths.

1 Introduction

In recent years many papers have been devoted to the study of photonic crystals *i.e.* periodic structures in which electromagnetic fields cannot propagate for some range of wave lenghts and directions [17], [9], [6], [5]. Band gaps have been exhibited numerically for some peculiar periodic structures [12], [13], [14], [11], [18].

Real experiments use, of course, samples which are not infinite periodic structures but slabs or films. So theoretically it is interesting to study not only the spectrum corresponding to these samples but also their scattering properties. Common believe is that the waves whose frequencies are inside the gap for the infinite structure, will be totally reflected by the slab regardless of their direction, if the width of the slab is sufficently large. We can also ask if for other frequencies waves can be almost totally reflected for some partial set of directions. The aim of this paper is to solve a model giving us a response to these questions.

In a first attempt we will not look at the electromagnetic situation with Maxwell equations but at the Schrödinger equation. So we will study hamiltonians corresponding to an infinite number of scatterers distributed over several planes, each of them forming a bidimensional lattice.

Each individual scatterer is described by a point interaction potential, which models an interaction of slow particles with a short range potential supported by a region much smaller than the wave length. Such a kind of model has been introduced by Bethe and Peierls [4] and Thomas [16] and studied in mathematically rigorous way by Berezin and Faddeev [3] and Grossman, Høegh-Krohn, and Mebkhout [8]. A detailed review has been given in the book by Albeverio *et al.* [1]. This simplification brings the advantage that the scattering by a unique interaction centre is explicitly known and simple. The scattered wave function at a point $\vec{x}$ corresponding to the incident plane wave $e^{i\vec{k}\vec{x}}$ is a spherical wave $\frac{e^{i\vec{k}\vec{x}}}{4\pi\alpha-ik}\frac{e^{ik|\vec{x}-\vec{y}|}}{4\pi|\vec{x}-\vec{y}|}$, where $\vec{y}$ is the position of the point scatterer and α is a parameter, positive or negative, which measures the strength of the interaction. As it is proven in [2] the point interaction is the limit in the norm resolvent sense of $\lambda(\epsilon)\epsilon^{-2}V((\vec{x}-\vec{y})/\epsilon)$, where $\lambda(\epsilon)$ is real analytic near the origin with $\lambda(0)=1$ and $V(x)$ gives a resonant state of energy 0.

Infinite number of scatterers whose potentials are zero–range potentials have been studied earlier in [10], [15], in particular the study of the spectrum corresponding to a layer can be found in [1].

In chapter 2, we give the expression for the generalized eigenfunctions. They are the sum of the incident plane wave, several scattered plane waves with different wave vectors linked with the two dimensional reciprocical lattice vectors and surface waves which decrease exponentially as the distance to the layers increase. The amplitudes of the plane waves *i.e.* the transmission and reflection coefficients can be expressed by series which are fairly convergent.

In chapter 3, we will give the numerical values for the transmission coefficients for some range of energies in the case of one layer. Playing with the strength of the individual scatterer α, we will see that the most intriguing effect is that for a small interval of α around the value -0.25, the reflection is close to 1 for wavelengths larger than twice the distance between the scatterers.

In chapter 4, we will give the numerical results for the amplitudes of the resulting scattered plane waves corresponding to 10 planes for several strengths of the individual scatterer.

2 The formalism

The point scatterers are located on N two dimensional parallel planes. Each layer is defined by a vector y_ν, $(\nu = 1, ...N)$, which fixes the seed, and by two independant vectors a_2 and a_3 whose coordinates are $(0, a_\ell^2, a_\ell^3) \in \mathbb{R}^3$, $\ell = 2, 3$. So the point scatterers in the ν^{th} plane are located on the two–dimensional lattice $\Lambda_{2,\nu} = \{y|y = y_\nu + m_2a_2 + m_3a_3 \in \mathbb{R}^3, m = (m_2, m_3) \in \mathbb{Z}^2\}$. As in [1], we denote by Λ_2 the set of vectors $m_2a_2 + m_3a_3 \in \mathbb{R}^3$, $(m_2, m_3) \in \mathbb{Z}^2$ and by $Y = \bigcup_\nu \Lambda_{2,\nu}$ the set of scatterers.

One denotes by γ, γ_n, or γ_{n_2,n_3} a vector of the two–dimensinal reciprocical lattice Γ_2, $\gamma = \gamma_n = \gamma_{n_2,n_3} = n_2 b_2 + n_3 b_3, (n_2, n_3) \in \mathbb{Z}^2$ where vectors b_2, b_3 satisfy $b_i a_j = 2\pi\delta_{ij}$.

Hamiltonian $-\Delta_{\alpha,Y}$ is defined in [1], where it is shown that it can be decomposed in a direct integral of self–adjoint operators $-\Delta_{\alpha,Y}(\theta)$ acting on $H(\theta)$ which is the set of functions $\phi(x_1, x_2, x_3)$ belonging to $L^2(\mathbb{R}, \mathbb{S}_2)$, ($\mathbb{S}_2$ is the two dimensional unit cell, $x_2 a_2 + x_3 a_3 \in \mathbb{S}_2$, if $0 < x_2, x_3 < 1$; $\mathbb{S}_2$ is called $\hat{\Gamma}_2$ in [1]) and such that $\phi(x_1, 1, 1) = e^{i(\theta_2+\theta_3)}\phi(x_1, 0, 0)$, where $\theta = (\theta_2, \theta_3)$ belongs to the two dimensional Brillouin zone B_2, called $\hat{\Lambda}_2$ in [1].

The reduced resolvent $(-\Delta_{\alpha,Y}(\theta) - z)^{-1}$ is given in [1], page 213 (1.6.23):

$$(-\Delta_{\alpha,Y}(\theta) - z)^{-1} = g_{\sqrt{z}}(\theta) + \sum_{j,j'} (\Gamma_{\alpha,Y}(\sqrt{z}, \theta))^{-1}_{jj'} g_{\sqrt{z}}(. - y'_j, \theta) g_{\sqrt{z}}(. - y_j, \theta), \quad (2.1)$$

where $g_{\sqrt{z}}(x, \theta) = \frac{1}{2}\sum_\gamma \frac{e^{-\sqrt{|\gamma+\theta|^2 - z}|x_1|}}{\sqrt{|\gamma+\theta|^2 - z}} e^{i(\gamma+\theta)x_\parallel}$, see (1.6.12) p. 211 in [1] and

$$\Gamma_{\alpha,Y}(\sqrt{z}, \theta)) = (\alpha\delta_{j,j'} - g_{\sqrt{z}}(y_j - y'_j, \theta))^N_{j,j'=1},$$

see (1.6.19) p. 212 in [1].

Then one can show that the Green function of $\Delta_{\alpha,Y}$ can be expressed as

$$G(x, y, z) = \frac{e^{i\sqrt{z}|x-y|}}{4\pi|x - y|} + \int_B d\theta \sum_{j,j'} (\Gamma_{\alpha,Y}(\sqrt{z}, \theta))^{-1}_{jj'} g_{\sqrt{z}}(x - y'_j, \theta) g_{\sqrt{z}}(y - y_j, \theta). \quad (2.2)$$

We calculate then $G(x, k, z) = \int G(x, y, z)e^{iky}dy$, $k = (k_1, k_\parallel) \in \mathbb{R}^3$ and define $\Psi(x, k) = \lim_{\epsilon\to 0} i\epsilon G(x, k, k^2 + i\epsilon)$.

One obtains

$$\begin{aligned}
\Psi(x, k) &= e^{ikx} + \frac{1}{2}\sum_\nu \left(\sum_{\nu'} \left[\hat{\Gamma}(|k|, k_\parallel - \gamma')^{-1}\right]_{\nu,\nu'} e^{iky_{\nu'}} \right) \\
&\times \left(\sum_{\gamma, |\gamma - k_\parallel| < |k|} \frac{e^{i\sqrt{k^2 - (\gamma - k_\parallel)^2}|x_1 - y_{\nu 1}|}}{\sqrt{k^2 - (\gamma - k_\parallel)^2}} e^{i(k_\parallel - \gamma)(x_\parallel - y_{\nu_\parallel})} \right. \\
&+ \left. \sum_{\gamma, |\gamma - k_\parallel| > |k|} \frac{e^{-\sqrt{(\gamma - k_\parallel)^2 - k^2}|x_1 - y_{\nu 1}|}}{\sqrt{(\gamma - k_\parallel)^2 - k^2}} e^{i(k_\parallel - \gamma)(x_\parallel - y_{\nu_\parallel})} \right),
\end{aligned} \quad (2.3)$$

where γ' is the reciprocical lattice vector such that $k_\parallel - \gamma'$ belongs to B_2. One can easily verify that $\Psi(x, k)$ is a solution of the Schrödinger equation:
$-\Delta_{\alpha,Y}\Psi(x, k) = k^2\Psi(x, k)$.

Now we want to establish the link in between (2.3) and a possible extension of the formula given in [1], p. 135 (1.5.1) for the scattering wave corresponding to a finite number of centers. This last one is written there as:

$$\psi_{\alpha,Y}(x) = e^{ikx} + \sum_j \left(\sum_{j'} [\Gamma_{\alpha,Y}(k)^{-1}]_{jj'} e^{iky_{j'}} \right) \frac{e^{i|k||x-y_j|}}{4\pi\,|x-y_j|}, \tag{2.4}$$

where $\Gamma_{\alpha,Y}(k)$ is a matrix whose elements are

$$\begin{aligned} [\Gamma_{\alpha,Y}(k)]_{jj'} &= \left(\alpha - \frac{i|k|}{4\pi}\right)\delta_{jj'} - \tilde{G}_k(y_j - y_{j'}) \\ Y &= \{y_1, y_2, \ldots\} \\ \tilde{G}_k(x) &= \begin{cases} \frac{e^{i|k||x|}}{4\pi|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}. \end{aligned}$$

This function is the sum of an incident plane wave and spherical waves centered at the scatterers.

In our case index j becomes a double index (ν, m) and the sums become infinite. If we denote by $f_{\nu m}(k) = \sum_{\nu' m'} [\Gamma_{\alpha,Y}(k)^{-1}]_{(\nu m),(\nu' m')} e^{iky_{\nu' m'}}$ the amplitude of the spherical scattered wave centered at $y_\nu + m_2 a_2 + m_3 a_3$ and by $u_{\nu m} = e^{iky_{\nu m}}$, we can express $u_{\nu m}$ in a condensed form: $u_{\nu m} = (\Gamma f)_{\nu m}$. We define now the functions $\hat{u}_\nu(\theta)$ and $\hat{f}_\nu(\theta)$ by

$$\begin{aligned} \hat{u}_\nu(\theta) &= \sum_m u_{\nu m} e^{-i\theta\lambda_m}, \qquad \theta \in B_2 \\ \hat{f}_\nu(\theta) &= \sum_m f_{\nu m} e^{-i\theta\lambda_m}. \end{aligned}$$

Using the translation invariance of the operator Γ with respect to the translations by vectors belonging to Λ_2, we can write:

$$\hat{u}_\nu(\theta) = \sum_{\nu'} \hat{\Gamma}(\theta)_{\nu,\nu'} \hat{f}_{\nu'}(\theta), \qquad \nu, \nu' = 1, 2, \ldots, N$$

where

$$\hat{\Gamma}(\theta)_{\nu,\nu'} = \sum_m \Gamma(y_\nu - y_{\nu'} + \lambda_m, 0) e^{-i\lambda_m \theta}.$$

Here we suppose that the sum exists. We will discuss this point later.
Inverting the $N \times N$ matrix $\hat{\Gamma}(\theta)$, one deduces

$$\hat{f}_\nu(\theta) = \sum_{\nu'} \left[\hat{\Gamma}(\theta)^{-1}\right]_{\nu,\nu'} \hat{u}_{\nu'}(\theta).$$

As

$$u_{\nu' m} = e^{ik(y_{\nu'} + \lambda_m)} = e^{iky_{\nu'}} e^{ik_\parallel \lambda_m}$$

by Poisson formula we get:

$$\hat{u}_{\nu'}(\theta) \;=\; e^{iky_{\nu'}}\sum_m e^{ik_{\|}\lambda_m}e^{-i\theta\lambda_m} \;=\; e^{iky_{\nu'}0}\sum_n \delta(\theta - k_{\|} - \gamma_n)$$

then

$$f_{\nu m}(k) \;=\; \int_{\hat{\Lambda}_2} e^{i\theta\lambda_m}\sum_{\nu'}\left[\hat{\Gamma}(\theta)^{-1}\right]_{\nu,\nu'}\hat{u}_{\nu'}(\theta)d\theta = \; e^{ik_{\|}\lambda_m}\sum_{\nu'}\left[\hat{\Gamma}(k_{\|})^{-1}\right]_{\nu,\nu'}e^{iky_{\nu'}}.$$

So the extension of (2.4) to the layer problem should be:

$$\psi_{\alpha,Y}(k,x) \;=\; e^{ikx} + \sum_{\nu}\left(\sum_{\nu'}\left[\hat{\Gamma}(k_{\|})^{-1}\right]_{\nu,\nu'}e^{iky_{\nu'}}\right)\sum_m e^{ik_{\|}\lambda_m}\frac{e^{i|k||x-y_\nu-\lambda_m|}}{4\pi|x-y_\nu-\lambda_m|}. \quad (2.5)$$

If in [1], p. 214, the Poisson type formulas (1.6.26) are still valid when $Im(k) = 0$, then $\hat{\Gamma}(k_{\|})_{\nu,\nu'}$ coincide with $\Gamma_{\alpha,Y}(|k|, k_{\|} - \gamma')$ and the sum over m in (2.5) coincides with the sum over γ in (2.3). So finally $\psi_{\alpha,Y}(x,k)$ coincides with the generalized eigenfunction defined from the Green function, $\Psi_{\alpha,Y}(x,k)$. From this fact we can interpret $\sum_{\nu'}\left[\hat{\Gamma}(k, k_{\|}-\gamma')^{-1}\right]_{\nu,\nu'}e^{iky_{\nu'}}$ in (2.3) as the amplitude $f_{\nu m}$ of the spherical scattered wave at $y_\nu + m_2a_2 + m_3a_3$.

Now consider several particular cases: if there is only one layer ($N = 1$) the amplitude of the wave scattered by the center located at $y_1 + \lambda_m$ is

$$f_{1m} = e^{ik_{\|}(y_1+\lambda_m)}\frac{1}{\alpha + \lim_{\omega\to\infty}\left(\frac{\omega}{4\pi} - \sum_{\gamma,|k_{\|}+\gamma|<\omega}\frac{i}{2\sqrt{k^2-(k_{\|}+\gamma)^2}}\right)}. \quad (2.6)$$

If moreover $k_{\|} = 0$ then f_{1m} is independent of m, as it is clear from symmetry considerations, and it depends only on $|k|$

$$\begin{aligned}\frac{1}{f(|k|)} \;&=\; -\frac{i}{2}\left(\sum_{\gamma,|\gamma|<|k|}\frac{1}{\sqrt{k^2-\gamma^2}}\right) - \\ &\quad \frac{1}{2}\lim_{\omega\to\infty}\left(\sum_{|k|<|\gamma|<\omega}\frac{1}{\sqrt{\gamma^2-k^2}} - \frac{\omega}{2\pi}\right) + \alpha.\end{aligned}$$

In this case, the generalized eigenfunction has the expression

$$\begin{aligned}\psi_{\alpha,Y}(x) \;&=\; e^{ikx} + \frac{i}{2}\frac{\hat{f}(|k|)}{k}e^{i|k_\perp||x^1|} + \frac{i}{2}\hat{f}(|k|)\sum_{0<|\gamma|<k}\frac{e^{i\sqrt{k^2-(\gamma)^2}|x^1|}}{\sqrt{k^2-\gamma^2}}e^{i\gamma\tilde{x}} + \\ &\quad \frac{\hat{f}(|k|)}{2}\sum_{|\gamma|>k}\frac{e^{-\sqrt{(\gamma)^2-k^2}|x^1|}}{\sqrt{\gamma^2-k^2}}e^{i\gamma\tilde{x}}.\end{aligned}$$

In the case $k_{||} = 0$ the wavefunction is the sum of plane waves with a wave vector whose first component is $\sqrt{k^2 - \gamma^2}$ (for all the γ such that $|\gamma| < |k|$) and of surface waves decreasing exponentially with exponent $\sqrt{\gamma^2 - k^2}$, ($|\gamma| > k$).

We denote by $A_{n_1 n_2}$ the amplitude of the plane wave $e^{i\sqrt{k^2 - |\gamma_{n_1 n_2}|^2} x^1}$. So

$$
\begin{aligned}
A_{0,0} &= 1 + \frac{i}{2}\frac{f(|k|)}{|k|} \\
A_{0,1} = A_{0,-1} = A_{0,1} = A_{-1,0} &= \frac{i}{2}\frac{f(|k|)}{\sqrt{k^2 - 4\pi^2}} \\
&\vdots \\
A_{n_1 n_2} = \ldots = A_{n_2 n_1} &= \frac{i}{2}\frac{f(|k|)}{\sqrt{k^2 - 4(n_1^2 + n_2^2)\pi^2}}
\end{aligned}
$$

Remark 1: If $k_{||} = 0$, because of the behavior of $f(|k|)$ near $|k| = |\gamma_{n_1 n_2}|$ one deduces that

$$
\begin{aligned}
A_{n_1 n_2}(|\gamma_{n_1 n_2}|_+) &= \frac{i}{2} 2i\sqrt{k^2 - |\gamma_{n_1 n_2}|^2}\frac{1}{\sqrt{k^2 - |\gamma_{n_1 n_2}|^2}} = -1 \\
A_{n_1 n_2}(|\gamma_{n_1' n_2'}|) &= 0 \qquad \text{for} \qquad |\gamma_{n_1' n_2'}| > |\gamma_{n_1 n_2}|.
\end{aligned}
$$

Remark 2: If $k_{||} = 0$, for $|k| < 2\pi$ one notices that the imaginary part of $\frac{1}{f(|k|)}$ is simply $-\frac{i}{2|k|}$, while for each value of $|k|$ it exists an α negative, such that the real part of $\frac{1}{f(|k|)}$ is zero. It can be noticed numerically that the real part of $\frac{1}{f(|k|)}$ is almost constant for $0 < |k| < 2.5$ and that for $\alpha = -0.25$ the real part of $\frac{1}{f(|k|)}$ is close to 0. So for $0 < |k| < 2.5$, $f(|k|)$ is close to $2i|k|$ in this range, then the transmission coefficient A_{00} is close to 0, and the reflection coefficient is close to 1.

3 Numerical calculation for one plane

In fig. 1 we present the amplitude of the scattered spherical waves $f(|k|, k_{||})$ for different values of α ($\alpha = -1, -0.25, 1$) in the case the incident wave vector is perpendicular to the plane of scatterers and in the case the wave vector is tilted. The zeroes of $f(|k|, k_{||})$ are determined by $k^2 - (k_{||} + \gamma)^2 = 0$; this implies that they coincide with absolute values of vectors of the reciprocical lattice when k is perpendicular to the plane of scatterers.

In fig. 2 we represent the amplitudes of the different plane waves. In the case the incident wave vector is perpendicular to the plane of scatterers and if $|k| < 2\pi$, there is only one scattered wave, its momentum is k and $|A_{00}|$ coincides with the amplitude of the transmitted wave. For $2\pi < |k| < 2\sqrt{2}\pi$ there are four plane waves more, with identical amplitudes $A_{1,0} = A_{0,-1} = A_{0,1} = A_{-1,0}$. If k is tilted, this "degeneracy" is generally destroyed, *i.e.* plane waves begin to appear at different values of $|k|$, that are solutions of $k^2 - (k_{||} + \gamma)^2 = 0$. $|A_{n_1,n_2}| = 1$ at these values of k (*cf.* Remark 1 of previous section).

As was stated in Remark 2 we verify that for $\alpha = -0.25$ the transmission is very small for a wide range of $|k|$ $(0 < |k| < \pi)$. This phenomenon is noticeable also for α in an interval around -0.25: $(-0.3, -0.2)$. Let us remind here that point interactions are smaller as $|\alpha| \to \infty$. $\alpha = -0.25$ corresponds to an attractive interaction larger than those corresponding to $\alpha = \pm 1$. ($\alpha = \pm\infty$ corresponds to a null interaction).

To be more confident with the numerical calculations we looked for a conservation law. In fact the flux of the current J has to be 0 through a closed surface. Taking a cylinder whose basis is the direct lattice cell and its length an arbitrary L, we calculate the flux of J through the surface of this cylinder. We verify this conservation law numerically and find it is fulfilled with a very good precision, the error is smaller than $< 10^{-14}$.

4 Numerical calculation for N planes

Here we only consider the case when the distance between the planes is constant, the individual α are all equal and $y_\nu^2 = y_\nu^3 = 0$ for all ν. We choose the distance equal to 1 and 5, $\alpha = -1, -0.25, 1$, and the number of planes $N = 10$.

Remark 1: For $\alpha = -0.25$ the amplitudes of the spherical waves on each scatterer are larger than in the case where there is only one plane of scatterers. For two planes the maxima of $|f_1(k)|$ and $|f_2(k)|$ exceed 20.

Remark 2: For a given direction of the incident wave vector, as the number of planes increases, the transmission coefficients decrease in some intervals. When $N = 10$ they are close to zero in some intervals. This fact is reminiscent of the behavior of the transmission coefficient in one dimension with a finite number of δ-functions (see [1] p. 275). There, as the number of scatterers tends to infinity the values of $|k|$ for which the transmission coefficient becomes close to zero correspond to energies in the gaps of the spectrum of the Kronig-Penney operator. These values are multiples of the half of the reciprocal lattice vector lenght of the one dimensional crystal.

As can be seen in fig. 4, when we fix the direction of the incident wave vector one observes a similar fact, A_{00} is small for regular values of $|k|$. When $k_{||} = 0$, these values correspond to half of the lenght of the reciprocal lattice vectors whose direction is perpendicular to the planes. In particular we observe that in the interval $[0, 2\pi]$ there are five times more minima in the case the distance in between the planes is five times larger. But here the corresponding energies and the gaps in the spectrum are no more related, since in the three dimensional infinite crystal there is at most one gap (see [1] p. 192). This means that in general, if we change the direction of the incident wave vector, the energies at which the transmission coefficient is small will change.

In fig. 3 we present the values for the transmission coefficients for several directions of the incident wave vector. We can observe that each curve for A_{00} presents pronounced dips in some wavelenght intervals, and there is one dip which is common to all the directions considered.

We have calculated the gap for $\alpha = -0.25$ it corresponds to the interval of energies $(-8.515, 8.19)$ and we can see that the common dip is under the value 2.7 which is near $\sqrt{8.19} = 2.86$. So there is a coincidence between the common dip energies and the gap of the three–dimensional crystal.

Besides this interval it exist other energy intervals for which the reflection is important only for incident wavevectors in a small solid angle.

5 Conclusion

As we have seen, we understand the relations between the transmission coefficients for a finite number of planes and the spectral properties of the infinite crystal. As expected for values of α close to -0.25 the reflection is close to 1 for the energies in the 3–dimensional gap, for all the incident directions. What is surprising is that the reflection is quite important even for an unique plane. That is probably a discovery which can be of practical importance. To be close to possible experiments, in a forthcoming paper we will study the case, the number of scatterers in each plane is large but finite and we will compare it to the limit case we have studied here.

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Figure captions

Figure 1 Absolute value of spherical waves $|f(|k|)|$ are shown for three values of scatterers strength $\alpha = -1, -0.25, 1$. The plane of scattering centres forms a square lattice of period 1 as described in sec. 2. The first row corresponds to the situation when the incident plane wave is perpendicular to the plane of scatterers and the second row to the incident plane wave with the vector k tilted ($\theta = \pi/3$ and $\phi = \pi/5$). In both cases $|f|$ is independent of m, $|f_m| = |\tilde{f}(|k|, k_{\|})|$. The dotted lines mark the positions of $|\gamma_{0,1}| = 2\pi$ and $|\gamma_{1,1}| = 2\sqrt{2}\pi$ in the upper row, while in the lower row they are determined by $k^2 - (k_{\|} + \gamma)^2 = 0$.

Figure 2 The first row shows the dependence of the amplitudes of scattered plane waves $|A_{0,0}|$, $|A_{0,1}|$, and $|A_{1,1}|$ on $|k|$ for three values of scatterers strengths $\alpha = -1, -0.25, 1$, for an incident plane wave perpendicular to the same plane of scatterers as in fig. 1. The second row shows the dependence of the amplitudes of scattered plane waves $|A_{0,0}|$ on $|k|$ for three values of scatterers strengths $\alpha = -1, -0.25, 1$, for an incident plane wave whose direction with respect to the plane of scatterers is given by $\theta = \pi/3$ and $\phi = \pi/5$.

Figure 3 Scattering by $N = 10$ planes regularly spaced with spacing 1. The first row shows, for an incident plane wave perpendicular to the planes, the dependence of the amplitudes of scattered plane waves $|A_{0,0}|$ on $|k|$ for three values of scatterers strengths $\alpha = -1, -0.25, 1$. The second row shows, the dependence of the amplitudes of scattered plane waves $|A_{0,0}|$ on $|k|$ and on several directions of the incident plane wave, (($\theta = 0, \phi = 0$) the full line, ($\theta = \pi/3, \phi = 0$) the dashed line, ($\theta = 2\pi/5, \phi = 0$) the dash–dotted line, and ($\theta = 49\pi/100, \phi = 0$) the dotted line) for three values of scatterers strengths $\alpha = -1, -0.25, 1$.

Figure 4 Scattering by $N = 10$ planes regularly spaced with spacing 1. The dependence of the amplitudes of scattered plane waves $|A_{0,0}|$ on $|k|$ and on several directions of the incident plane wave, (($\theta = 2\pi/5, \phi = 0$) the full line, ($\theta = 2\pi/5, \phi = \pi/8$) the dashed line, ($\theta = 2\pi/5, \phi = \pi/6$) the dash–dotted, and ($\theta = 2\pi/5, \phi = \pi/4$) the dotted line) for three values of scatterers strengths $\alpha = -1, -0.25, 1$.

Figure 5 The dependence of the amplitudes of scattered plane waves $|A_{0,0}|$ on $|k|$ and on the spacing between the planes. The incident plane wave is perpendicular to a slab made of 10 planes regularly spaced with spacings 1 and 2, respectively. The upper figure shows cosine of Bloch phase of the infinite periodic slab (crystal) of the spacing 1 (the full line), 2 (the dashed line). The dashed lines in the middle and lower figures shows the $|A_{0,0}|$ for one plane of scatterers. The strength of scatterers is $\alpha = -1$.