On the cosmological implications of the Calogero conjecture

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Abstract

We discuss the implications of the Calogero hypothesis on the "cosmic origin of quantization" [1], and of the variation of h with time it implies, for cosmology. We focus on two issues: (a) how the relation between redshift and distance would be modified; and (b) how the observations on the cosmic background radiation, and the estimation they give of the time elapsed since its production, would be interpreted in the light of Calogero hypothesis. We also check that the hypothesis is compatible with the present bounds on the variations of fundamental constants and with the observed blackbody distribution of the cosmic background radiation.

1 Introduction

In a recent paper, Calogero [1] reconsidered the "universal background noise" which, according to Nelson's stochastic mechanics [2, 3, 4] is at the basis of quantum behaviour. Calogero considered the possible *physical* origin of this noise, pointing his attention to gravitational interactions with distant masses in the Universe (a different mechanism is considered in [5]); he argued that in this way one obtains a "prediction" for the Planck constant given by

$$h \;\approx\; \alpha\, G^{1/2}\, m^{3/2}\, [R(t)]^{1/2} \tag{1}$$

with α a numerical constant. Here G is the Newton gravitational constant, m is the mass of the hydrogen atom, considered as the basic unit of mass in the Universe, and $R(t)$ represents the radius of the Universe, or better of the part of it accessible to gravitation interactions, at time t.

With the rough estimate $R(t_0) \simeq 10^{28}$ cm for the present value of R and taking $\alpha = 1$, equation (1) gives the remarkable estimate $h \approx 6 \cdot 10^{-26}\, \mathrm{cm}^2 \mathrm{g}\, \mathrm{s}^{-1}$.

It should be noted that R varies with time and therefore (unless G and/or m also vary with time and a miracolous cancellation occurs) the Calogero conjecture (1) entails that h is also varying with time.

If we accept that G and m are truly constant (which we do), then it follows that according to Calogero

$$h \equiv h(t) = A \sqrt{R(t)} = h_0 \sqrt{a(t)} \tag{2}$$

with A a dimensional constant. Here we have denoted by $h_0 = h(t_0)$ the present value of the Planck constant, and by $a(t)$ the cosmic scale factor (see below), normalized so that $a(t_0) = 1$.

It is maybe appropriate to mention that Calogero conjecture should not necessarily be seen in the frame of Nelson's stochastic mechanics [2, 3, 4]. Indeed, one could indipendently of this enquire on what is the statistical effect of interaction with distant masses in the Universe on the motion of a particle; the computation by Calogero [1] shows that this is expected to be of the same order of magnitude[1] as quantum mechanical effects; I thank prof. Calogero for stressing this point of view to me (personal communication).

We want to discuss the consequences of this conjecture for cosmology, and in particular: (a) on the relation between redshift and distance of the emitting objects[2] and on the issue of the age of the Universe; and (b) on the cosmic background radiation and again on its implication for the age of the Universe.

It should be stressed that the physical discussion by Calogero [1] has no reason to apply as is to the primaeval epoch of the Universe; in particular, in a radiation-dominated Universe one should not consider the interaction with matter without considering interaction with radiation. We will however extend Calogero formula back to the crossover time when radiation and matter were in equilibrium and when the cosmic background radiation had its origin, and examine what the consequences are on the equilibrium temperature and on the age of the cosmic background radiation.

The *plan of the paper* is as follows. In the next short section 2 we note explicitely that the Calogero conjecture is compatible with the (rather strict) observational bounds on the variation of fundamental constants. In section 3 we briefly recall the solution of the Einstein equation for a flat Universe with zero cosmological constant, and the relation between time and distance for radiation in this case. In section 4 we analyze how the Calogero conjecture would affect the relation between distance and observed frequency of electromagnetic radiation (restricting for the sake of simplicity to a well defined atomic transition, e.g. the first spectral line of Hydrogen). We pass then to examine the cosmic background radiation issue; in section 5 we discuss how the Calogero conjecture would affect the adiabatic expansion of this; in section 6 we check that the

[1] If one wants, one can then think of two sources of "randomness" in dynamics: on the one side quantum mechanical effects, on the other gravitational interaction as analyzed by Calogero; it seems however more economical to think that the two are actually related.

[2] In a previous note [6] we have considered this issue in the small redshift approximation and using standard coordinates; in this note we will not assume Z to be small and will use comoving coordinates.

Calogero conjecture is not in contrast with the black body initial distribution of the cosmic background radiation, and in section 7 we discuss how it would affect the cosmological information we extract from the present state of the cosmic background radiation, focusing in particular on the age of the Universe and on the temperature at which the cosmic background radiation was emitted.

2 Bounds on the variation of the fine structure constant and the Calogero conjecture

In view of such a radical conjecture as the one relating h to the radius of the Universe, it is natural – and needed – to wonder if this is compatible with the existing observations and first of all with those on the bounds on the possible variation of fundamental constants. It should be stressed that the very concept of variations of a dimensional constant is not so well defined (after all, our definition of time and length units depend on h), so that we can have precise bounds only on non-dimensional constants. Among these, one directly related to h is the fine structure constant $\alpha = 2\pi e^2/(hc) \simeq 1/137$. Notice that accepting e and c to be constant we have that $|\dot{\alpha}/\alpha| = |\dot{h}/h|$.

Bounds on $|\dot{\alpha}/\alpha|$ have been studied; the most stringent one [7] is $|\dot{\alpha}/\alpha| < 5.0 \cdot 10^{-17}\mathrm{yr}^{-1} \simeq 2 \cdot 10^{-24}\mathrm{s}^{-1}$.

From (2) we have that $|\dot{h}/h| = |\dot{R}/R|$, and with the present model of expansion we have, as mentioned above, $\dot{R} = D_0/\sqrt{R}$; thus, recalling that $D_0 = \sqrt{2MG/R_0^3}$, we have $(\dot{R}/R) = \sqrt{2GM}/R^3$.

Even with the "pessimistic" estimates $M = 4 \cdot 10^{62}\mathrm{g}$ and $R_0 = 10^{26}\mathrm{cm}$, we get $(\dot{R}/R)_0 \approx 10^{-50}\,\mathrm{s}^{-1}$.

Thus the Calogero conjecture is, at least in the case of a flat Universe with zero gravitational constant, well compatible with experimental bounds on the time variation of the fine structure constant.

We would like to explicitely remark that a discussion of other "compatibility with experiments" conditions for the Calogero conjecture requires a very careful analysis: indeed, a varying h would modify the Physics of all kind of phenomena, so that such an analysis could not accept the standard interpretation of virtually any observational data (as we will see when discussing the cosmic background radiation).

3 Expansion of a flat Universe.

Our considerations will depend on the large scale dynamics of the Universe. In this section we will briefly recall the explicit time dependence of quantities of interest in the case of a flat Universe; for more detail see e.g. [8, 9]. We will be interested in the time evolution of the cosmic scale factor $a(t)$, and on the relation between the distance r of a source and the time $\tau(r)$ needed for the radiation it emits to reach us.

The analysis of expansion (assumed to be uniform) is better performed in comoving coordinates [8, 9], i.e. by considering a static model with a varying metric. This variation is embodied in a cosmic scale factor[3] $a(t)$: that is, all distances satisfy $x(t) = a(t)\sigma_0$ (we will denote distances in comoving coordinates as σ). In particular, $R(t) = a(t)R_0$.

The evolution of $a(t)$ is governed by the Einstein equations [8, 9], which for a homogeneous and isotropic matter-dominated Universe are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_{\rm m}}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{3}$$

where $\rho_{\rm m}$ is the mass density of the Universe, Λ the cosmological constant, and k a parameter equal to (1,0,-1) for a (closed, flat, open) universe.

We will assume $\Lambda = 0$ and consider the flat case[4] $k = 0$. Notice that $\rho_{\rm m}(t) = \rho_{\rm m}^0/a^3(t)$ [with $\rho_{\rm m}^0 \equiv \rho_{\rm m}(t_0)$]. Writing $D_0^2 = (8/3)\pi G\rho_{\rm m}^0$, eq.(3) reads then $\dot{a}(t) = D_0/\sqrt{a(t)}$, which is readily solved by separation of variables to give (the c_i will denote arbitrary constants) $a(t) = [(3/2)(D_0 t + c_1)]^{2/3}$. With $D_1 = (3/2)D_0$, $c_2 = (3/2)c_1$, this is rewritten as $a(t) = (D_1 t + c_2)^{2/3}$. By definition, $a(t_0) = 1$, so that $(D_1 t_0 + c_2) = 1$, which yields $c_2 = 1 - D_1 t_0$ and hence $a(t) = [1 - D_1(t_0 - t)]^{2/3}$. We rewrite this final result as

$$a(t_0 - \tau) = (1 - D_1\tau)^{2/3} , \tag{4}$$

with $D_1 = \sqrt{6\pi G\rho_{\rm m}^0} = 3\sqrt{(GM)/(2R_0^3)}$, M the total mass of the Universe and R_0 its present radius.

Let us now consider the relation between r and $(t_0 - t) \equiv \tau$. In a static Universe, radiation emitted at a distance r would reach us after a time $\tau = r/c$; in an expanding Universe this relation should be modified to take into account the expansion, i.e. the variation of the metric.

Thus we have

$$\int_{t_0-\tau}^{t_0} \frac{c}{a(t)}\, dt = \sigma . \tag{5}$$

Using (3), an elementary integration gives $(3c/D_1)[1 - (1 - D_1\tau)^{1/3}] = \sigma$ and therefore $(1 - D_1\tau) = [1 - (D_1/3c)\sigma]^3$; that is,

$$\tau = \frac{1}{D_1}\left(1 - [1 - (D_1/3c)\sigma]^3\right) . \tag{6}$$

4 Relation between observed frequency and the expansion of the Universe.

We consider an expanding Universe and denote by $a(t)$ the cosmic scale factor, normalized as to have $a(t_0) = 1$, where t_0 is our present time. We want

[3] The cosmic scale factor will always be normalized so that at the present time t_0 it satisfies $a(t_0) = 1$.

[4] The recent observational results on the cosmic background radiation [10] have strenghtened this scenario.

to analyze the relation between speed and distance in view of the Calogero conjecture[5].

If radiation is emitted by an object at rest in comoving coordinates (i.e. such that its motion in standard coordinates is due only to the Universe expansion) at time $t_{\rm em}$ with frequency $\nu_{\rm em}$, and is observed by us at time t_0, due to the change in the metric and thus is the cosmic scale factor, it is observed with a different frequency $\nu_{\rm ob}$. The shift in frequency is measured by the quantity

$$Z \ := \ \frac{\lambda_{\rm ob} - \lambda_{\rm em}}{\lambda_{\rm em}} \ \equiv \ \frac{\nu_{\rm em}}{\nu_{\rm ob}} - 1 \ . \tag{7}$$

One also refers to Z as the redshift; we will focus on the ratio $\nu_{\rm em}/\nu_{\rm ob}$.

As all distances, and in particular the wavelength λ of radiation, change according to the cosmic scale factor, we have at once that $\lambda_{\rm em}/a(t_{\rm em}) = \lambda_{\rm ob}/a(t_0)$, and therefore [using $\lambda\nu = c$ and $a(t_0) = 1$], denoting also by τ the time needed for the radiation to reach us,

$$\nu_{\rm ob} \ = \ \nu_e \, a(t_{\rm em}) \ \equiv \ \nu_{\rm em} \, a(t_0 - \tau) \ . \tag{8}$$

Thus, see (4), we have

$$\nu_{\rm ob} \ = \ (1 - D_1 \tau)^{2/3} \ \nu_{\rm em} \ , \tag{9}$$

and the redshift $Z(\tau)$ of the radiation emitted at a time $t = t_0 - \tau$ is given by $Z(\tau) = (1 - D_1\tau)^{-2/3} - 1$. We stress that this result does not depend on the behaviour of h in any way, but only on assuming an evolution of the Universe governed by Einstein equations (3) with $k = 0$ and $\Lambda = 0$.

We can use (6), which again depends only on the Einstein equations and on assuming $k = 0, \Lambda = 0$, in order to express $Z(\tau)$ in terms of the distance of the emitting object. Expressing the latter in comoving coordinates as σ, and writing $D_2 = D_1/(3c)$, we have then $\nu_{\rm ob} = (1 - D_2\sigma)^2 \nu_{\rm em}$; this also means that σ can be obtained in terms of the ratio between observed and emitted frequency as

$$\sigma \ = \ (1/D_2) \ \left[1 - \sqrt{\nu_{\rm ob}/\nu_{\rm em}} \right] \ . \tag{10}$$

It is now time to distinguish between the standard case and the one devised by the Calogero conjecture.

Let us focus, for the sake of concreteness, on a well specific radiation, i.e. the one corresponding to the first spectral line of hydrogen. We know then that

$$\nu_{\rm em} \ = \ \frac{B}{h^3} \tag{11}$$

where, with standard notation, $B = (3/8) m_{\rm e} e^4 \pi^2$. We denote the value of (11) with the known value (according to Calogero, the known present value) of h by ν_0. We write $\nu_{\rm ob} = \beta\nu_0$ and $\nu_{\rm em} = \mu\nu_0$.

[5]This analysis was conducted in standard coordinates in [7]; at present we will perform it in comoving coordinates.

According to standard quantum mechanics, $\nu_{\rm em} = \nu_0$ (i.e. $\mu = 1$) and thus (8) reads $\beta = a(t_0 - \tau)$.

According to Calogero conjecture, due to the radiation being emitted at different time and thus with a different value of $h(t)$, we have $\nu_{\rm em} \neq \nu_0$ (i.e. $\mu \neq 1$) and thus (8) reads

$$\beta \;=\; \mu\, a(t_0 - \tau) \;. \tag{15}$$

For the first hydrogen spectral line we would have (with t_0 the present time, $t_{\rm em}$ the emission time) $\nu_{\rm em} = B/h^3(t_{\rm em}) = [B/h^3(t_0)][h(t_0)/h(t_{\rm em})]^3$, and therefore, see (2),

$$\mu \;=\; \frac{\nu_{\rm em}}{\nu_o} \;=\; \left[\frac{h(t_0)}{h(t_{\rm em})}\right]^3 \;=\; [a(t_{\rm em})]^{-3/2} \;. \tag{13}$$

[We have used the normalization of $a(t)$ giving $a(t_0) = 1$]. In the following we write, for ease of notation, $\tau = t_0 - t_{\rm em}$; from (4) we have then

$$\mu \;=\; (1 - D_1\tau)^{-3/2} \;. \tag{14}$$

Let us now go back to the relation between observed frequency and comoving distance of the emitting object. According to standard quantum mechanics $\mu = 1$, and thus (10) gives

$$\sigma_{\rm q}(\beta) \;=\; \frac{1}{D_2}\,\left(1 - \sqrt{\beta}\right) \;. \tag{15}$$

According to Calogero conjecture μ is given by (14); using (6) eq.(10) reads then $D_2\sigma_{\rm cal} = [1 - (1 - D_2\sigma)^{3/4}\sqrt{\beta}]$, and therefore

$$\sigma_{\rm cal}(\beta) \;=\; \frac{1}{D_2}\,\left(1 - \beta^2\right) \;. \tag{16}$$

Notice that comparing (15) and (16) we have $(\sigma_c(\beta)/\sigma_{\rm q}(\beta)) = (1 + \sqrt{\beta})(1 + \beta)$. For small redshift, i.e. $\beta = 1 - \varepsilon$, we have $\sigma_{\rm cal}/\sigma_{\rm q} \;\simeq\; 4 + O(\varepsilon^2)$.

It is well known that the relation between recession speed and distance is expressed by the Hubble law $v = H(t)r$, where $H(t)$ is the Hubble constant.

Thus for small redshift – i.e. small distance[6] – we still have, accepting the Calogero conjecture, an approximately linear relation between recession speed and distance, but with a different value (by a factor four) of the Hubble constant.

5 Adiabatic cooling of cosmic background radiation

The cosmic background radiation (CBR) is a precious source of information on the early stages of the Universe and on its age. We want now to discuss how the Calogero conjecture would affect the interpretation of experimental data on the CBR.

[6]This is obviously the case for which the estimation of the distance is more reliable.

The total energy of a photon gas at temperature T is given by

$$E_0 \; = \; \frac{\alpha V}{h^3} T^4 \tag{17}$$

where V is the volume occupied by the gas and $\alpha = 16\pi^5 k^4/(15c^3)$. Its entropy is given by

$$S = (4/3)(\alpha/h^3)T^3 V \tag{18}$$

In a adiabatic expansion, $T = T(t)$ and $V = V(t)$ change so to keep S constant. In a flat Universe, $V = (4/3)\pi R^3$. Therefore, in the standard setting (i.e. with h constant) we have with T_0 the temperature of the photon gas at t_0 [and using as always the normalization $a(t_0) = 1$],

$$T_{\rm q}(t) \; = \; \frac{T_0}{a(t)} \; . \tag{19}$$

However, if we consider Calogero conjecture, h should also be thought as a time-dependent quantity, see (2). Thus (18) reads in this case

$$S \; = \; \eta \, T^3(t) \, R^{3/2}(t) \tag{20}$$

where obviously $\eta = (4/3)^2\pi/A^3$, with A defined in (2) above. Hence, using again $R(t) = a(t)R(t_0)$ and $a(t_0) = 1$, we conclude that according to Calogero conjecture, in the adiabatic expansion of a photon gas (as the CBR) the temperature dependence on time – i.e. on the cosmic scale factor $a(t)$ of the expanding Universe – is given by

$$T_{\rm cal}(t) \; = \; \frac{T_0}{\sqrt{a(t)}} \; . \tag{21}$$

Thus, the Calogero conjecture would imply a much slower cooling of the Universe than the standard (h constant) theory. In section 7 we make a quantitative comparison and discuss the consequences on the age of CBR and therefore of the Universe.

6 The black body character of the CBR and Calogero conjecture

In the previous section, we have seen how the temperature of the CBR would change with time according to the Calogero conjecture. However, temperature is not the only characteristic of the CBR: this is also characterized by having a black body (BB) distribution, and this character is of fundamental importance for cosmology.

Indeed, we know that with the standard laws of Physics (i.e. with h constant) the CBR, having a BB distribution at the time t^E of transition between a radiation-dominated and a matter-dominated Universe[7], would be observed

[7] In the fololowing we refer to this just as "transition time", and similarly for the corresponding temperature.

nowadays as still having a BB distribution, albeit with a lower temperature. This is indeed the case, and this observation confirms that in the origin (at t^E) the CBR was emitted with a BB distribution.

If this was not the case many aspects of cosmology would be at stake, so that the Calogero conjecture can avoid to be discarded on the basis of fundamental cosmological considerations only if it retains this fundamental aspect of CBR. This is actually the case, as we check in this section by an elementary computation; this will follow the standard computation ensuring the CBR retains its BB character in the standard case, see e.g. [8].

In a BB distributed radiation at temperature T, the number of photons with frequencies between ν and $\nu + d\nu$ contained in a volume V at time t is given by

$$dN(t) \ = \ \frac{8\pi}{c^3}\ V\ \frac{\nu^2}{\exp\left[(h\nu)/(\kappa T)\right] - 1}\ d\nu \ . \tag{22}$$

In an expanding Universe, at the later time t_0 [for which $a(t) = 1$] the volume will have expanded to $V_0 = V/a^3(t)$; the frequency ν will have redshifted to[8] $\nu_0 = \nu a(t)$ [so that $d\nu_0 = a(t) d\nu$]; and the temperature T will have decreased to T_0. Acoording to Calogero conjecture, also h will have changed to h_0.

We have thus to express (22) in terms of the new quantities ν_0, V_0, T_0 and, in the light of Calogero conjecture, with a different value of h, i.e. h_0. Using the scaling law (21) for $T(t)$ and (2) for $h(t)$, we have that the argument of the exponential is just

$$[(\sqrt{a(t)} h_0\, a^{-1}(t) \nu_0) \,/\, (\kappa\, T_0 / \sqrt{a(t)})] \ \equiv \ [(h_0 \nu_0)/(\kappa T_0)] \ ; \tag{23}$$

it is also easy to check that the a factor in front of V, ν^2 and $d\nu$ cancel out, so that in the end

$$dN(t_0) \ = \ \frac{8\pi}{c^3}\ V_0\ \frac{\nu_0^2}{\exp\left[(h_0\nu_0)/(\kappa T_0)\right] - 1}\ d\nu_0 \ . \tag{24}$$

In this way, we obtain that these same photons still have a BB distribution, although with a different temperature $T_0 = \sqrt{a(t)} T$ (and of course a different value of the Planck constant h); that is, the Calogero conjecture does not affect the BB distribution of the CBR.

7 The age of the Universe from CBR in the light of Calogero conjecture

Let us first of all recall the standard theory for the age of Universe and the transition temperature as "measured" from the present temperature of the CBR [8, 9]. We will denote by $\mathrm{d_r}$ and $\mathrm{d_m}$ the present observed average radiation density and matter density of the Universe, and by T_0 the present temperature of the CBR.

[8] No confusion should arise with the ν_0 used in section 4 above.

As we have seen above, the radiation temperature would go as $T(t) = T_0/a(t)$; therefore the radiation density evolves according to $\rho_{\rm r}(t) = {\rm d_r}/a^4(t)$. On the other hand, the matter density obviosuly evolves as $\rho_{\rm m}(t) = {\rm d_m}/a^3(t)$.

Thus the equilibrium between radiation and matter densities, $\rho_{\rm m} = \rho_{\rm r}$, is obtained at the equilibrium time $t_{\rm q}^E$, identified by

$$a(t_{\rm q}^E) \;=\; \frac{\rm d_r}{\rm d_m} \; . \tag{25}$$

The temperature at this stage is obtained according to (19) to be

$$T_{\rm q}^E \;=\; \left(\frac{\rm d_m}{\rm d_r}\right) T_0 \; ; \tag{26}$$

as for the time $t_{\rm q}^E := t_0 - \tau_{\rm q}$, we have from (4) that

$$\tau_{\rm q} \;=\; \frac{1}{D_1}\left[1 - \left(\frac{\rm d_r}{\rm d_m}\right)^{3/2}\right] \; . \tag{27}$$

Let us now examine how these results would be changed according to Calogero conjecture, i.e. accepting (2).

First of all we note that the energy of radiation is given by (17); setting $a(t_0) = 1$ and using (2) and (21) we get then $E(t) = [E(t_0)/a^{7/2}(t)]$. The radiation density $\rho_{\rm r}(t) = E(t)/c^2$ is hence given by

$$\rho_{\rm r}(t) \;=\; \frac{\rm d_r}{a^{7/2}(t)} \; ; \tag{28}$$

the formula for $\rho_{\rm m}$ is still given by $\rho_{\rm m} = {\rm d_m}/a^3(t)$. Hence the equilibrium $\rho_{\rm r} = \rho_{\rm m}$ is obtained for $t_{\rm cal}^E$ such that

$$a(t_{\rm cal}^E) \;=\; \left(\frac{\rm d_r}{\rm d_m}\right)^2 \; . \tag{29}$$

Using again (21) for the evolution of $T(t)$, we obtain that

$$T_{\rm cal}^E \;=\; \left(\frac{\rm d_m}{\rm d_r}\right) T_0 \; . \tag{30}$$

The time $t_{\rm cal}^E := t_0 - \tau_{\rm cal}$ can be obtained using (4) to express a in terms of τ; in this way we get

$$\tau_{\rm cal} \;=\; \frac{1}{D_1}\left[1 - \left(\frac{\rm d_r}{\rm d_m}\right)^3\right] \; . \tag{31}$$

Let us finally compare the results obtained in the standard and in the Calogero scenarios.

First of all, we notice that the transition temperature T^E is equal in the two cases, see (26) and (30). This can be understood considering that at this temperature we have equilibrium between radiation and matter, and the conjecture we are considering only affects the cooling of radiation, not the matter expansion.

Let us now consider the age of the Universe, or more precisely the time elapsed from the transition, according to the standard and the Calogero scenarios. It turns out, perhaps surprisingly in view of the radical difference between (19) and (21), that the Calogero conjecture barely affects this.

Indeed, from (28) and (31) we have that

$$\frac{\tau_{\rm cal}}{\tau_{\rm q}} \ = \ \frac{1-(\mathrm{d_r}/\mathrm{d_m})^3}{(\mathrm{d_r}/\mathrm{d_m})^{3/2}} \ = \ 1+(\mathrm{d_r}/\mathrm{d_m})^{3/2} \ . \tag{32}$$

The current estimate for $\mathrm{d_r}$ and $\mathrm{d_m}$ are [8, 9]: $\mathrm{d_r} \approx 3 \cdot 10^{-34}\,\mathrm{g\,cm^{-3}}$; $\mathrm{d_m} \approx 3 \cdot 10^{-31}\,\mathrm{g\,cm^{-3}}$. The latter could change by a large factor due to non-visible matter, but this would increase the value of $\mathrm{d_m}$ and thus make the ratio $(\mathrm{d_r}/\mathrm{d_m})$ even smaller.

Thus, the difference between $\tau_{\rm cal}$ and $\tau_{\rm q}$ turns out to be extremely small on the cosmological scale, $\tau_{\rm cal} - \tau_{\rm q} \approx 3 \cdot 10^{-5}\, \tau_{\rm q}$.

However, the value of $\alpha := a(t^E)$, and thus the scale of the Universe, at the transition time turns out to be quite different in the two scenarios: indeed we have $\alpha_{\rm cal} = (\alpha_{\rm q})^2$. Thus, according to the Calogero conjecture the Universe would have been much denser at the transition time; this can be put in relation with the different value of h at that time.

8 Conclusions.

I have considered the consequence of the Calogero conjecture on the time variation (on cosmological time scales) of the Planck constant h for two fundamental aspects of cosmology, i.e. (a) the relation between observed frequency of radiation and distance of the emitting source, and (b) the estimates of the age of the Universe based on the present densities of radiation and matter and the present temperature of the CBR.

The interpretation of observational data would be changed in the light of Calogero conjecture, but remarkably we observed that: (a) the linear relation between recession speed and distance would be preserved for objects with small redshifts (the majority of observable ones and the only ones for which our distance estimates are reliable), although with a different proportionality constant; (b) the temperature at which the transition originating the CBR took place would not be affected, the age of the CBR would be only marginally affected, but the density of the Universe at the transition would be greatly affected.

We also observed that the Calogero conjecture is compatible with the present bounds on the variation of the fine structure constant.

To conclude this short note, we recall that Calogero conjecture is not the first attempt to relate quantum and large scale phenomena; it was already observed by Dirac that the ratio

$$D := \frac{G/H}{h^3 \epsilon_0 / (m_p^3 e^2)} \tag{33}$$

built with macroscopic and microscopic quantities, has a value quite near to unity[9], despite the enormous differences in the order of magnitude of the quantities appearing in it, and wondered if this was some kind of miracolous coincidence or is hiding some not yet understood deep relation. In [1] Calogero suggested a mechanism which could be at the basis of this relation, and which would have as a consequence a time variation of h on cosmological timescales; in this note I have shown that Calogero conjecture is not ruled out by some fundamental cosmological observational data.

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[9] With the recent adjustement in the value of the Hubble constant H, estimated now to be $H \simeq 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, it turns out that $D \approx 1.02$.