Existence and regularity of weak solutions to the prescribed

mean curvature equation for a nonparametric surface

P. Amster. M. Cassinelli M. C. Mariani and D.F. Rial
Departamento de Matemática, FCEyN-UBA

ABSTRACT:

We give conditions on the boundary data and the function h in order to obtain multiple solutions for the problem (1) below, using variational methods. We also study the regularity of the solutions.

Introduction

The prescribed mean curvature equation with Dirichlet condition for a nonparametric surface $X : \Omega \longrightarrow R^3$, $X(u,v) = (u,v,f(u,v))$ is the quasilinear partial differential equation

$$(1) \begin{cases} (1+f_v^2)f_{uu} + (1+f_u^2)f_{vv} - 2f_uf_vf_{uv} = 2h(u,v,f)\left(1+|\nabla f|^2\right)^{\frac{3}{2}} & \text{in } \Omega \\ f = g \quad \text{in } \partial\Omega \end{cases}$$

where Ω is a bounded domain in R^2, $h : \overline{\Omega} \times R \longrightarrow R$ is continuous and $g \in H^1(\Omega)$.

We call $f \in H^1(\Omega)$ a weak solution of (1) if $f \in g + H_0^1(\Omega)$ and for every $\varphi \in C_0^1(\Omega)$

$$\int_\Omega \left((1+|\nabla f|^2)^{-1/2}\nabla f \nabla\varphi + 2h(u,v,f)\varphi\right) dudv = 0$$

It is known that for the parametric Plateau's problem, weak solutions can be obtained as critical points of a functional (see [2], [5], [6], [7], [9], [10]).

The nonparametric case has been studied for $H = H(x,y)$ (and generally $H = H(x_1, ..., x_n)$ for hypersurfaces in R^{n+1}) by Gilbarg, Trudinger, Simon, Serrin, among other authors. It has been proved [12] that there exists a solution for any smooth boundary data if the mean curvature H' of $\partial\Omega$ satisfies:

$$H'(x_1, ..., x_n) \geq \frac{n}{n-1}|H(x_1, ..., x_n)|$$

for any $(x_1, ..., x_n) \in \partial\Omega$, and $H \in C^1(\overline{\Omega}, R)$ satisfying the inequality:

$$|\int_\Omega H\varphi| \leq \frac{1-\epsilon}{n}\int_\Omega |D\varphi|$$

for any $\varphi \in C_0^1(\Omega, R)$ and some $\epsilon > 0$. They also proved a non-existence result ([12], corollary 14.13): if $H'(x_1, ..., x_n) < \frac{n}{n-1}|H(x_1, ..., x_n)|$ for some $(x_1, ..., x_n)$ and the sign of H is constant, then for any $\epsilon > 0$ there exists $g \in C^\infty(\overline{\Omega})$ such that $\|g\|_\infty \leq \epsilon$ and that Dirichlet's problem is not solvable.

We remark that the solutions obtained in [12] are classical. In this work we find weak solutions of the problem by variational methods.

We will prove that for prescribed h there exists an associated functional to h, and under some conditions on h and g we will find that this functional has a global minimum in a convex subset of $H^1(\Omega)$, which provides a weak solution of (1). We denote $H^1(\Omega)$ the usual Sobolev space, [1].

The associated variational problem

Given a function $f \in C^2(\Omega)$, the generated nonparametric surface associated to this function is the graph of f in R^3, parametrized as $X(u,v) = (u, v, f(u,v))$.

The mean curvature of this surface is

$$h(u,v,f) = \frac{1}{2}\frac{Ef_{vv} - 2Ff_{uv} + Gf_{uu}}{(1 + f_u^2 + f_v^2)^{\frac{3}{2}}}$$

where E, F and G are the coefficients of the first fundamental form [3], [8].

For prescribed h, weak solutions of (1) can be obtained as critical points of a functional:

PROPOSITION 1 *Let $J_h : H^1(\Omega) \longrightarrow R$ be the functional defined by*

$$J_h(f) = \int_\Omega \Big((1 + |\nabla f|^2)^{1/2} + H(u,v,f)\Big)\, dudv \tag{2}$$

where $H(u,v,z)=\int_0^z 2h(u,v,t)dt$ *. Then (1) is the Euler Lagrange equation of (2).*

REMARK *If* $f\in T=g+H_0^1(\Omega)$ *is a critical point of* J_h *, then* f *is a weak solution of (1).*

Proof For $\varphi\in C_0^1(\Omega)$, integrating by parts we obtain:

$$dJ_h(f)(\varphi)=2\int_\Omega\left(\frac{1}{2}\frac{Ef_{vv}-2Ff_{uv}+Gf_{uu}}{(1+f_u^2+f_v^2)^{\frac{3}{2}}}-h(u,v,f)\right)\varphi dudv$$

Behavior of the functional J_h

In this section we will study the behavior of the functional J_h restricted to T . For simplicity we write $J_h(f)=A(f)+B(f)$, with

$$A(f)=\int_\Omega(1+|\nabla f|^2)^{1/2}dudv,\qquad\qquad B(f)=\int_\Omega H(u,v,f)dudv.$$

We'll assume that h is bounded.

LEMMA 2 *The functional* $A:T\longrightarrow R$ *is continuous and convex.*

Proof Continuity can be proved by a simple computation. Let $a,b\geq 0$ such that $a+b=1$. By Cauchy inequality, it follows that

$$\sqrt{1+|\nabla(af+bf_0)|^2}\leq a\sqrt{1+|\nabla f|^2}+b\sqrt{1+|\nabla f_0|^2}$$

and convexity holds.

REMARK *As* A *is continuous and convex, then it is weakly lower semicontinuous in* T *.*

LEMMA 3 *The functional* B *is weakly lower semicontinuous in* T *.*

Proof Since h is bounded, we have that

$$|H(u,v,z)|\leq c|z|+d$$

¿From the compact immersion $H_0^1(\Omega)\hookrightarrow L^1(\Omega)$ and the continuity of Nemytski operator associated to H in $L^1(\Omega)$, we conclude that B is weakly lower semicontinuous in T (see [4] and [11]).

Weak solutions as critical points of J_h

Let us assume that $g \in W^{1,\infty}$, and consider for each $k > 0$, the following subset of T:

$$\overline{M}_k = \{f \in T : \qquad \|\nabla(f-g)\|_\infty \leq k\}.$$

$\overline{M}_k$ is nonempty, closed, convex, bounded, then it is weakly compact.

REMARK: *as $g \in W^{1,\infty}$, taking $p > 2$ we obtain, for any $f \in \overline{M}_k$:*

$$\|f-g\|_p \leq c\|\nabla(f-g)\|_p$$

Then, by Sobolev imbedding, $\|f-g\|_\infty \leq c_1\|f-g\|_{1,p} \leq \overline{c}k$ for some constant $\overline{c}$. We deduce that $f \in W^{1,\infty}$ and $f(\Omega) \subset K$ for some fixed compact $K \subset R$. Thus, the assumption $\|h\|_\infty < \infty$ will not be needed.

Let ρ be the slope of J_h in $\overline{M}_k$ defined by:

$$\rho(f_0, \overline{M}_k) = \sup\{dJ_h(f_0)(f_0 - f); f \in \overline{M}_k\}$$

(see [6] and [9]), then the following result holds:

LEMMA 4 *If $f_0 \in \overline{M}_k$ verifies*

$$J_h(f_0) = \inf\{J_h(f) : f \in \overline{M}_k\}$$

then $\rho(f_0, \overline{M}_k) = 0$.

Proof

$$dJ_h(f_0)(f-f_0) = \lim_{\varepsilon \longrightarrow 0} \frac{J_h(f_0 + \varepsilon(f-f_0)) - J_h(f_0)}{\varepsilon} = \lim_{\varepsilon \longrightarrow 0} \frac{J_h((1-\varepsilon)f_0 + \varepsilon f) - J_h(f_0)}{\varepsilon}.$$

When $0 < \varepsilon < 1$ we have that $(1-\varepsilon)f_0 + \varepsilon f \in \overline{M}_k$, and then $dJ_h(f_0)(f_0 - f) \leq 0$ for all $f \in \overline{M}_k$. As $dJ_h(f_0)(f_0 - f_0) = 0$, we conclude that $\rho(f_0, \overline{M}_k) = 0$.

REMARK:

Being J_h weakly semicontinuous and $\overline{M}_k$ a weakly compact subset of T, J_h achieves a minimum f_0 in $\overline{M}_k$. By lemma 4, $\rho(f_0, \overline{M}_k) = 0$.

As in [6], if f_0 has zero slope, we call it a ρ-critical point. The following result gives sufficient conditions to assure that if f_0 is a ρ-critical point, then it is a critical point of J_h.

THEOREM 5

Let $f_0 \in \overline{M}_k$ such that $\rho(f_0, \overline{M}_k) = 0$, and assume that one of the following conditions holds:

i) $dJ_h(f_0)(f_0 - g) \geq 0$

ii) $\|\nabla(f_0 - g)\|_\infty < k$

Then $dJ_h(f_0) = 0$.

<u>*Proof*</u>

As $\rho(f_0, \overline{M}_k) = 0$, we have that $dJ_h(f_0)(f_0 - f) \leq 0$ and then $dJ_h(f_0)(f_0 - g) \leq dJ_h(f_0)(f - g)$ for any $f \in \overline{M}_k$.

We'll prove that $dJ_h(f_0)(\varphi) = 0$ for any $\varphi \in C_0^1$. Let $\widetilde{\varphi} = \frac{k\varphi}{2\|\nabla\varphi\|_\infty}$, then $\pm\widetilde{\varphi} + g \in \overline{M}_k$, and then $dJ_h(f_0)(f_0 - g) \leq \pm dJ_h(f_0)(\widetilde{\varphi})$.

Let us suppose that $dJ_h(f_0)(\widetilde{\varphi}) \neq 0$, then $dJ_h(f_0)(f_0 - g) < 0$.

If i) holds, we immediately get a contradiction. On the other hand, if ii) holds, there exists $r > 1$ such that $g + r(f_0 - g) \in \overline{M}_k$. Then $dJ_h(f_0)(f_0 - g) \leq r dJ_h(f_0)(f_0 - g)$, a contradiction.

EXAMPLES

Let us assume that $\int_\Omega \frac{\nabla(f-g)\nabla g}{\sqrt{1+|\nabla f|^2}} dudv \geq 0$ *for any* $f \in \overline{M}_k$. *Then condition i) of theorem 5 is fulfilled for example if:*

a) $|h(u,v,z)| \leq c(z - g(u,v))_+$ *for every* $(u,v) \in \Omega$, $z \in R^3$, *for some constant* c *small enough.*

b) $\int_\Omega h(u,v,f)(f-g)dudv \geq 0$ *for every* $f \in \overline{M}_k$. *As a particular case, we may take* $h(u,v,z) = c(z - g(u,v))$ *for any* $c \geq 0$.

c) $h(u,v,z) = -c(z - g(u,v))$ *for some* $c > 0$ *small enough.*

Indeed, in all the examples the inequality $dJ_h(f)(f - g) \geq 0$ *holds for any* $f \in \overline{M}_k$, *since*

$$dJ_h(f)(f-g) = \int_\Omega \left(\frac{\nabla f \nabla(f-g)}{\sqrt{1+|\nabla f|^2}} + 2h(u,v,f)(f-g) \right) dudv =$$

$$\int_\Omega \left(\frac{|\nabla(f-g)|^2}{\sqrt{1+|\nabla f|^2}} + 2h(f-g) \right) dudv + \int_\Omega \frac{\nabla(f-g)\nabla g}{\sqrt{1+|\nabla f|^2}} dudv \geq$$

$$\int_\Omega \left(\frac{|\nabla(f-g)|^2}{\sqrt{1+|\nabla f|^2}} + 2h(f-g) \right) dudv.$$

Then the result follows immediately in example b). In examples a) and c), being $\|\nabla(f-g)\|_\infty \leq k$ *we can choose* $\widetilde{k}$ *such that* $\sqrt{1+\|\nabla f\|_\infty^2} \leq \tilde{k}$ *. Then*

$$\int_\Omega \left(\frac{|\nabla(f-g)|^2}{\sqrt{1+|\nabla f|^2}} + 2h(u,v,f)(f-g) \right) dudv \geq \int_\Omega \left(\frac{|\nabla(f-g)|^2}{\tilde{k}} - 2c(f-g)^2 \right) dudv$$

$$\geq \frac{1}{\tilde{k}} \|\nabla(f-g)\|_2^2 - 2cc_1^2 \|\nabla(f-g)\|_2^2 = (\frac{1}{\tilde{k}} - 2cc_1^2) \|\nabla(f-g)\|_2^2$$

where c_1 *is the Poincaré's constant associated to* Ω.

Thus, the result holds for $c \leq \frac{1}{2\tilde{k}c_1^2}$ *.*

Remark *As it happens in the preceding examples, it can be proved that if* $dJ_h(f)(f-g) \geq 0$ *for any* $f \in \overline{M}_k$ *, then* g *is a weak solution of (1). Indeed, if* $dJ_h(g) \neq 0$ *, from theorem 5 it follows that* $\rho(g, \overline{M}_k) > 0$ *. As* J_h *achieves a minimum in every* $\overline{M}_k$ *, we may take* $k \geq k_n \longrightarrow 0$ *, and* f_n *such that* $\rho(f_n, \overline{M}_{k_n}) = 0$ *. As* $\overline{M}_{k_n} \subset \overline{M}_k$ *, condition i) in theorem 5 holds and then* $dJ_h(f_n) = 0$ *. It is immediate that* $f_n \longrightarrow g$ *in* $W^{1,\infty}$ *and then it follows easily that* $dJ_h(g) = 0$ *.*

Furthermore, for constant g *we can see that if* $dJ_h(f)(f-g) \geq 0$ *for any* $f \in \overline{M}_k$ *, then* g *is a global minimum of* J_h *in* $\overline{M}_k$ *: let us define* $\varphi(t) = J_h(tf + (1-t)g)$ *, then* $\varphi'(t) = dJ_h(tf + (1-t)g)(f-g)$ *. As* $0 \leq dJ_h(tf+(1-t)g)(tf+(1-t)g-g) = tdJ_h(tf+(1-t)g)(f-g)$ *it follows that* $J_h(f) - J_h(g) = \varphi(1) - \varphi(0) = \varphi'(c) \geq 0$ *.*

Multiple Solutions

In this section we study the multiplicity of weak solutions of (1). Let us consider:

$$\overline{N}_k = \{f \in \overline{M}_k \cap H^2 : \|\frac{\partial^2 f}{\partial x_i \partial x_j}\|_2 \leq k\}$$

$\overline{N}_k$ is a nonempty, closed, bounded, and convex subset of T , therefore $\overline{N}_k$ is weakly compact.

Then we obtain the following theorem, which is a variant of the Mountain Pass Lemma:

Theorem 6

Let $f_0 \in \overline{N}_k$ be a local minimum of J_h and assume that $J_h(f_1) < J_h(f_0)$ for some $f_1 \in \overline{N}_k$. Let

$$c = \inf_{\gamma \in \Gamma} \sup_{t \in [0,1]} J_h(\gamma(t))$$

where $\Gamma = \{\gamma \in C([0,1], \overline{N}_k) : \gamma(0) = f_0, \gamma(1) = f_1\}$. Then there exists $f \in \overline{N}_k$ such that $J_h(f) = c$ and $\rho(f, \overline{N}_k) = 0$.

We remark that f is not a local minimum of J_h. This kind of f is called an unstable critical point.

The proof of theorem 6 follows from theorem 3 in [6] and lemmas 7-8-9 below.

Lemma 7

The functional J_h is $C^1(\overline{N}_k)$.

__*Proof*__ Let $f, f_0 \in \overline{N}_k$. Then

$$|dJ_h(f)(\varphi) - dJ_h(f_0)(\varphi)| \leq \|\varphi\|_{H_0^1} \left(\left\| \frac{\nabla f}{\sqrt{1+|\nabla f|^2}} - \frac{\nabla f_0}{\sqrt{1+|\nabla f_0|^2}} \right\|_2 + \|N_h(f_0) - N_h(f)\|_2 \right)$$

where N_h is the Netmyski operator associated to h. Being

$$\left\| \frac{\nabla f}{\sqrt{1+|\nabla f|^2}} - \frac{\nabla f_0}{\sqrt{1+|\nabla f_0|^2}} \right\|_2 \leq \left\| \sqrt{1+|\nabla f_0|^2}\nabla f - \sqrt{1+|\nabla f|^2}\nabla f_0 \right\|_2 \leq \kappa \|f_0 - f\|_{H_0^1}$$

and $N_h : L^2 \longrightarrow L^2$ continuous, the result holds.

Lemma 8 *The slope ρ is H^1- continuous.*

__*Proof*__ Let $f_n \in \overline{N}_k$ such that $f_n \longrightarrow f_0$ in H_0^1. For $\epsilon > 0$ we take $g_n \in \overline{N}_k$ such that $\rho(f_n, \overline{N}_k) - \frac{\epsilon}{2} < dJ_h(f_n)(f_n - g_n)$. Then

$$\rho(f_n, \overline{N}_k) - \rho(f_0, \overline{N}_k) \leq dJ_h(f_n)(f_n - g_n) + \frac{\epsilon}{2} - dJ_h(f_0)(f_0 - g_n)$$

$$\leq \|dJ_h(f_n)\|_{(H_0^1)^*} \|(f_n - f_0)\|_{H_0^1} + \|dJ_h(f_n) - dJ_h(f_0)\|_{(H_0^1)^*} \|(f_0 - g_n)\|_{H_0^1} + \frac{\epsilon}{2} < \epsilon$$

for $n \geq n_0$. Operating in the same way with $\rho(f_0, \overline{N}_k) - \rho(f_n, \overline{N}_k)$, we conclude that $\rho(f_n, \overline{N}_k) \longrightarrow \rho(f_0, \overline{N}_k)$.

Lemma 9 (Palais Smale condition)

Let $(f_n)_{n\in N} \subset \overline{N}_k$ *such that* $\lim_{n\longrightarrow\infty} \rho(f_n, \overline{N}_k) = 0$. *Then* $(f_n)_{n\in N}$ *has a convergent subsequence in* $H_0^1(\Omega)$.

<u>*Proof*</u> As $f_n \in \overline{N}_k$, we may suppose that $f_n \longrightarrow f$ weakly. Let $\Psi_n = f_n - f$. We'll see that $\Psi_n \longrightarrow 0$. Indeed,

$$dJ_h(f_n)(\Psi_n) = \int_\Omega \left(\frac{\nabla f_n}{\sqrt{1+|\nabla f_n|^2}} \nabla\Psi_n + 2h(u,v,f_n)\Psi_n \right) dudv =$$

$$= \int_\Omega \frac{1}{\sqrt{1+|\nabla f_n|^2}} |\nabla\Psi_n|^2 \, dudv + \int_\Omega \frac{\nabla\Psi_n}{\sqrt{1+|\nabla f_n|^2}} \nabla f dudv +$$

$$+ \int_\Omega 2h(u,v,f_n)\Psi_n dudv.$$

Then for some constant c

$$c\,\|\nabla\Psi_n\|_2^2 \le \rho(f_n, \overline{N}_k) - \int_\Omega \frac{\nabla\Psi_n}{\sqrt{1+|\nabla f_n|^2}} \nabla f dudv - \int\limits_\Omega 2h(u,v,f_n)\Psi_n dudv$$

By Rellich-Kondrachov theorem $\Psi_n \to 0$ in $L^2(\Omega)$, and then

$$\left| \int_\Omega 2h(u,v,f_n)\Psi_n dudv \right| \le 2\,\|h\|_\infty |\Omega|^{\frac{1}{2}} \|\Psi_n\|_2 \to 0$$

and

$$\left| \int_\Omega \frac{\nabla\Psi_n}{\sqrt{1+|\nabla f_n|^2}} \nabla f dudv \right|$$

$$= \left| -\int_\Omega \frac{\Delta f}{\sqrt{1+|\nabla f_n|^2}} \Psi_n dudv - \int_\Omega \Psi_n \nabla \left(1+|\nabla f_n|^2\right)^{-\frac{1}{2}} \nabla f dudv \right|$$

$$\le \|\Delta f\|_2 \|\Psi_n\|_2 + \|\nabla f_n\|_\infty \|\nabla f\|_\infty \left\|D^2 f_n\right\|_2 \|\Psi_n\|_2 \longrightarrow 0.$$

Example

Now we'll show with an example that problem (1) may have at least three ρ*-critical points in* N_k*:*

Let $g = g_0$ *be a constant, and* $h(u,v,z) = -c(z-g_0)$ *for some constant* $c > 0$. *Then,* g_0 *is a minimum of* J_h *in* $\overline{M}_{k_1}$ *for* k_1 *small enough, and a local minimum in* M_k *for any* $k \ge k_1$.

Moreover, taking $\Omega = B_R$, $f(u,v) = g_0 + R^2 - (u^2+v^2)$, *it follows that*

$$J_h(f) - J_h(g_0) = 2\pi(o(R^3) - \frac{c}{6}R^6)$$

and taking $k = 2\sqrt{\pi}R$ *it holds that* $f \in \overline{N}_k$. *Hence, if* R *is big enough it follows that* g_0 *is not a global minimum in* $\overline{N}_k$. *Furthermore, we see that the proof of lemma 4 may be repeated in* $\overline{N}_k$, *and then the minimum of* J_h *in* $\overline{N}_k$ *is a* ρ*-critical point. From Theorem 6 there is a third* ρ*-critical point which is not a local minimum of* J_h .

Regularity

As we proved, problem (1) admits (for an apropiated $k > 0$) a weak solution in a subset $\overline{M}(k) = \{f \in T \quad / \quad \|\nabla(f-g)\|_\infty \le k\}$.

Let us consider $p > 2$, and $f_0 \in W^{2,p}(\Omega) \hookrightarrow C^1(\overline{\Omega})$ a weak solution of (1). Then $L_{f_0} f_0 = 2h(u,v,f_0)(1+\nabla f_0^2)^{3/2}$ in Ω where for any $f \in C^1(\overline{\Omega})$ $L_f : W^{2,p} \longrightarrow L^p$ is the strictly elliptic operator given by

$$L_f \phi = (1+f_v^2)\phi_{uu} + (1+f_u^2)\phi_{vv} - 2 f_u f_v \phi_{uv}$$

In order to prove the regularity of f_0, we study the equation (2) $L_{f_0}\phi = 2h(u,v,f_0)(1+\nabla f_0^2)^{3/2}$ in Ω , $\phi = g$ in $\partial\Omega$.

Proposition 10

Let us assume that $\partial\Omega \in C^{2,\alpha}$, $g \in C^{2,\alpha}$ *and* $h \in C^\alpha$ *for some* $0 < \alpha \le 1 - \frac{2}{p}$. *Then, if* $\phi \in W^{2,p}$ *is a strong solution of (2),* $\phi \in C^{2,\alpha}(\overline{\Omega})$.

Proof

By Sobolev imbedding $\phi \in C^{1,\alpha}(\overline{\Omega})$. Then $L_{f_0}\phi \in C^\alpha(\overline{\Omega})$ and the coefficients of the operator L_{f_0} belong to C^α . By theorem 6.14 in [8], the equation $Lw = L_{f_0}\phi$ in Ω , $w = g$ in $\partial\Omega$ is uniquely solvable in $C^{2,\alpha}(\overline{\Omega})$, and the result follows from the uniqueness in theorem 9.15 in [8].

Remark

As a simple consequence, we obtain that $f_0 \in C^{2,\alpha}(\overline{\Omega})$, *by the uniqueness in* $W^{2,p}$ *given by theorem 9.15 in [8].*

Corollary 11

Let us assume that $\partial\Omega \in C^{k+2,\alpha}$, $g \in C^{k+2,\alpha}$ *and* $h \in C^{k,\alpha}$ *for some* $0 < \alpha \le 1 - \frac{2}{p}$. *Then* $f_0 \in C^{k+2,\alpha}(\overline{\Omega})$.

Proof

It is immediate from Proposition 1 and theorem 6.19 in [8].

ACKNOWLEDGEMENT

The authors thank specially Prof. J.P.Gossez for the careful reading of the manuscript and his suggestions and remarks.

References

[1] Adams R. A., Sobolev Spaces. Academic Press, 1975.

[2] Brézis H. and Coron J.M., Multiple solutions of H- system and Rellich's conjeture. Comm. Pure Appl. Math. 37 (1984), 149 - 184.

[3] Do Carmo M., Differential Geometric of Curves and Surfaces. Prentice-Hall, 1976.

[4] Goldstein Costa D., Tópicos em Analise não Linear e Aplicacões as Ecuacões Diferenciais. I.M.PA., 1986.

[5] Hildebrandt S., On th Plateau problem for surfaces of constant mean curvature. Comm. Pure Appl. Math. 23 (1970), 97- 114.

[6] Lami Dozo E. and Mariani M.C., A Dirichlet problem for an H-system with variable H. Manuscripta Mathematica 81 (1993), 1 - 14.

[7] Lami Dozo E. and Mariani M.C., Solutions to the Plateau problem for the prescribed mean curvature equation via the mountain pass lemma. Studies un Applied Mathematics 96 (1996), 351 - 358.

[8] Osserman R., A survey of minimal surfaces, Van Nostrand Reinhold Company, 1969.

[9] Struwe M., Plateau's Problem and the Calculus of Variations. Math. Notes 35, Princeton University Press. Princeton 1989.

[10] Struwe M., Non uniqueness in the Plateau problem for surfaces of constant mean curvature, Arch. Rat. Mech. Anal. 93 (1986), 135-157.

[11] Vainberg M. M., Variational Methods in the Theory of Non Linear Operators, Holden-Day, San Francisco, 1964.

[12] Gilbarg, D. Trudinger, N. S. : Elliptic partial differential equations of second order, Springer- Verlag (1983).

P. Amster *, M. M. Cassinelli *, M. C. Mariani * and D. F. Rial

Dpto. de Matemática, Fac. de Cs. Exactas y Naturales, UBA. PAB I, Cdad. Universitaria, 1428. Capital, Argentina.

* CONICET

Address for correspondence:

Prof. M. C. Mariani, Dpto. de Matemática, Fac. de Cs. Exactas y Naturales, UBA. PAB I, Cdad. Universitaria, 1428. Capital, Argentina.

email:

mcmarian@dm.uba.ar