Non-existence of the Møller Morphism for the Spin Fermion Dynamical System.

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We give a precise non trivial counterexample to the existence of the Møller morphism for groups of *-automorphism describing a two-body interacting dynamical system. This fact invalidate the property of L^1-asymptotic abelianness which is a crucial ingredient in many developments of the non-stationary approach on the stability of KMS states. It is build on the spin fermion dynamical system, nevertheless it is typically what we should expect when we consider a generic case of a small system interacting with a reservoir.

1. INTRODUCTION

One of the main topic of quantum statistical mechanics is the study of equilibrium properties of a system involving a large number of particles. The algebraic approach characterize equilibrium states as a state over a $\mathbb{C}^*$-algebra of observables and the dynamic of the system as one-parameter group of $*$-automorphisms

$$A \to \tau_t(A) = \mathrm{e}^{\mathrm{i}tH} A \mathrm{e}^{-\mathrm{i}tH}.$$

In this picture the so-called KMS condition play an important role in the description of equilibrium states for infinite systems and even finite systems. Indeed, it uniquely determines the Gibbs state and it is taken to be the empirical definition of equilibrium. A lot of effort was been made in the study of KMS condition, one can found in the literature several characterization and developments e.g.[9]. In particular, the stability aspect of those states was been investigated within two different approaches. Namely, Given a two dynamical system where one is a perturbation of the other by P, how their KMS states are linked? The first approach is a stationary method due to Araki [3] and gives an expansion of the Ω^P, the perturbed KMS vector state in term of the free one Ω or more shortly $\Omega^P = \mathrm{e}^{(H+P)/2}\Omega$, using the modular theory. The second approach initiated by Robinson, [19], is time-dependent and more general. It makes the attempt to identify the perturbed equilibrium state as the evoluted free equilibrium state under the perturbed dynamic. This is somehow in contradiction with the physical expectation which is typically the improvement of ergodic properties when the interaction is turned on and it was been rigorously confirmed with the relatively recent results on return to equilibrium [14], [5]. The main ingredient behind the non-stationary approach is a sort of asymptotic abelianness of observables with respect to the evoluted ones, sometimes appearing in a stronger condition called L^1-asymptotic abelianness i.e:

$$\int_{-\infty}^{+\infty} ||[A, \tau_t(B)]|| dt < \infty.$$

The later condition has nice consequence, namely it allows the construction of Møller morphism

$$\gamma_\pm = \lim \tau_{-t}^P \circ \tau_t(A),$$

where A is any element of the algebra of observables and $\gamma_\pm$ is the morphism whom its dual maps KMS states of the perturbed dynamic to the KMS states of the unperturbed one.

In this note we show the non-existence of the Møller morphism for a simplified model and therefore the above condition does not hold. This is somehow what one should expect to happen even for more complicated examples. We make many restriction to keep simplified the proof with regard to its aim, nevertheless we believe that hypothesis as massive fermion, bounded interaction, zero temperature do not play a major role. In fact, the main obstacle is that the perturbed evolution and the free one isolated different subsystems which is reflected by the fact that their derivations have different eigenvalues. The similarity with quantum scattering theory is to notice and the idea of introducing an identification to construct wave operator could have an application in this framework.

2. THE SPIN-FERMION SYSTEM

We start by defining some notations and recall some basic objects related to fermionic Fock space and CAR representation. In Subsect. 2.2 we introduce Wick polynomials and give a sufficient condition for their boundedness. They enter on the expression of the unitary implementation of the perturbed time-evolution.

2.1. Fermionic Fock spaces

Let $\mathfrak{h}$ be a complex Hilbert space. Let $\Lambda^n(\mathfrak{h})$ denotes the anti-symmetric n-fold tensor product of $\mathfrak{h}$. The *fermionic Fock space* over $\mathfrak{h}$ is

$$\Lambda(\mathfrak{h}) := \bigoplus_{n\geq 0} \Lambda^n(\mathfrak{h}),$$

where $\Lambda^0(\mathfrak{h}) := \mathbb{C}$. We define the finite particle space $\Lambda^{\text{fin}}(\mathfrak{h})$ as the subspace of finite sums of vectors in $\Lambda^n(\mathfrak{h})$. Let Λ_n be the orthogonal projection from the usual tensor product $\otimes^n\mathfrak{h}$ into the subspace $\Lambda^n(\mathfrak{h})$. It acts in a pure tensor product state as follows:

$$\Lambda_n(f_1\otimes ...\otimes f_n) := \frac{1}{n!}\sum_{\pi\in S_n}\epsilon(\pi) f_{\pi_1}\otimes ...\otimes f_{\pi_n},\ \text{ where } f_i\in\mathfrak{h},\ \text{for } i=1,...,n.$$

The above sum is taken over all the permutations of n elements and $\epsilon(\pi)$ denotes the signature of those permutations. Let $\psi\in\Lambda^n(\mathfrak{h}), \phi\in\Lambda^m(\mathfrak{h})$, we denote by $\psi\wedge\phi$ the anti-symmetric vector $\Lambda_{n+m}\psi\otimes\phi$.

We recall the annihilation and creation distributions in the case where $\mathfrak{h} := L^2(\mathbb{R}^d)$, which are operator-valued tempered distributions. Let $S(\mathbb{R}^d)$ be the Schwartz space. Let $\Psi\in\Lambda^{\text{fin}}(\mathfrak{h})$:

$$(b(k)\Psi)^{(n)}(k_1,...,k_n) := (n+1)^{\frac{1}{2}}\,\Psi^{(n+1)}(k,k_1,...,k_n),$$

$$(b^*(k)\,\Psi)^{(n)}(k_1,...,k_n) := n^{-\frac{1}{2}}\sum_{j=1}^{n}(-1)^{j-1}\,\delta(k-k_j)\,\Psi^{(n-1)}(k_1,...,\hat{k}_j,...,k_n),$$

where $\hat{k}_j$ means that k_j is omitted and δ is the Dirac function. The annihilation and creation operators $b(h), b^*(h)$, $h\in\mathfrak{h}$ are defined by the following weak integrals

$$\begin{aligned} b(h) &:= \int b(k)\,\overline{h(k)}\,dk,\\ b^*(h) &:= \int b^*(k)\,h(k)\,dk. \end{aligned}$$

They satisfy the canonical anti-commutation relations,

$$\{b^*(h), b^*(g)\} = 0, \quad \{b(h), b(g)\} = 0, \tag{2.1}$$

$$\{b(h), b^*(g)\} = (h|g)\mathbb{1}, \qquad h, g \in \mathfrak{h}. \tag{2.2}$$

The so-called *Pauli principle* tell us that $b(h)^2 = b^*(h)^2 = 0$. This leads with (2.2) to the boundedness of creation-annihilation operators $||b^\sharp(h)|| = ||h||$.

We define the second quantization of a one particle operator A denoted by $\mathrm{d}\Gamma(A)$ as

$$\mathrm{d}\Gamma(A)_{|\Lambda^n(\mathfrak{h})} = \sum_{j=1}^{n} \mathbf{1} \otimes ... \otimes A \otimes ... \otimes \mathbf{1},$$

where the sum runs over A in the j^{th} position. The number operator N is what we get as second quantization of the identity. We mention that in general $\mathrm{d}\Gamma(A)$ are unbounded operators even if A are bounded and they are bounded if and only if A are trace class operators.

Let B be an operator acting on $\mathfrak{h}$. The functor operator $\Gamma(B)$ is

$$\Gamma(B) : \Lambda(\mathfrak{h}) \to \Lambda(\mathfrak{h}),$$

$$\Gamma(B)_{|\Lambda^n(\mathfrak{h})} := B^{\otimes(n)},$$

with the notation $B^{\otimes(n)} := B \otimes ... \otimes B$, n times.

2.2. Wick Polynomials

Let $w \in \mathcal{B}(\Lambda^p(\mathfrak{h}), \Lambda^q(\mathfrak{h}))$. We define the wick monomial with symbol w to be the operator $Wick(w)$ acting as following:

$$Wick(w) : \Lambda^{\mathrm{fin}}(\mathfrak{h}) \to \Lambda^{\mathrm{fin}}(\mathfrak{h}),$$

$$Wick(w)_{|\Lambda^n(\mathfrak{h})} := \frac{\sqrt{n!(n+q-p)!}}{(n-p)!}\, w \wedge \mathbb{1}^{\otimes(n-p)}, \text{ if } n \geq p,$$

otherwise it is equal to zero. A Wick polynomial is a finite sum of Wick monomials with symbols in $\mathcal{B}(\Lambda^{p_i}(\mathfrak{h}), \Lambda^{q_i}(\mathfrak{h}))$, $p_i, q_i \in \mathbb{N}$. We mention that a Wick monomial can be defined even if the symbol is unbounded operator, but we restrict our selves to bounded symbols. We have the following lemma.

Lemma 2.1. *We have*

$$\begin{aligned}
&i)\, \lambda \in \mathbb{C},\ Wick(\lambda) = \lambda\mathbb{1},\\
&ii)\, w \in \mathcal{B}(\mathfrak{h}),\ Wick(w) = \mathrm{d}\Gamma(w),\\
&iii)\, h \in \mathfrak{h},\ Wick(\langle h|) = b(h),\ Wick(|h\rangle) = b^*(h).\\
&iv)\, h_1, ..., h_n \in \mathfrak{h},\ Wick(|h_1 \wedge ... \wedge h_n\rangle) = b^*(h_1)...b^*(h_n),\\
&\qquad\qquad\qquad\qquad Wick(\langle h_1 \wedge ... \wedge h_n|) = b(h_n)...b(h_1).
\end{aligned}$$

Let $\mathcal{K}$ be an auxiliary complex Hilbert space. We extend the definition of Wick polynomials to be observables taking in account the degrees of freedom of $\mathcal{K}$. Hence, for the symbol $w \in \mathcal{B}(\mathcal{K} \otimes \Lambda^p(\mathfrak{h}), \mathcal{K} \otimes \Lambda^q(\mathfrak{h}))$, we define Wick monomial as follows:

$$\begin{aligned}
&Wick(w) : \mathcal{K} \otimes \Lambda^{\text{fin}}(\mathfrak{h}) \to \mathcal{K} \otimes \Lambda^{\text{fin}}(\mathfrak{h}),\\
&Wick(w)_{|\mathcal{K}\otimes\Lambda^n(\mathfrak{h})} := \frac{\sqrt{n!(n+q-p)!}}{(n-p)!}\ \mathbf{1}_{\mathcal{K}} \otimes \Lambda_{n+q-p}\ \ w \otimes \mathbf{1}^{\otimes(n-p)}.
\end{aligned}$$

Let summarize properties of the above extension of Wick polynomials in the following lemma.

Lemma 2.2. *Let* $w \in \mathcal{B}(\mathcal{K} \otimes \Lambda^p(\mathfrak{h}), \mathcal{K} \otimes \Lambda^q(\mathfrak{h}))$, $u \in \Lambda^n(\mathfrak{h})$, $v \in \Lambda^m(\mathfrak{h})$. *We have*

$$\begin{aligned}
&i)\, Wick(w)^* = Wick(w^*).\\
&ii)\, Wick(|u\rangle \wedge w \wedge \langle v|) = Wick(|u\rangle)\, Wick(w)\, Wick(\langle v|).
\end{aligned}$$

In the case $\mathfrak{h} := L^2(\mathbb{R}^d)$. Using the kernel theorem (see e.g. [23]), we know that

$$L(S(\mathbb{R}^{dp}), S^{'}(\mathbb{R}^{dq}, \mathcal{B}(\mathcal{K}))) \cong S^{'}(\mathbb{R}^{dp}) \hat{\otimes} S^{'}(\mathbb{R}^{dq}, \mathcal{B}(\mathcal{K})) \cong S^{'}(\mathbb{R}^{d(p+q)}, \mathcal{B}(\mathcal{K}))$$

Then $w \in \mathcal{B}(\mathcal{K} \otimes \Lambda^p(\mathfrak{h}), \mathcal{K} \otimes \Lambda^q(\mathfrak{h}))$ has a distribution kernel in $S^{'}(\mathbb{R}^{d(p+q)}, \mathcal{B}(\mathcal{K}))$. Hence, Wick monomials have the following weak integral representation:

$$Wick(w) = \int\ w(k_1, ..., k_p, k_1^{'}, ..., k_q^{'})\ b^*(k_1^{'})...b^*(k_q^{'}) \times b(k_1)...b(k_q)\ dkdk^{'}.$$

The above expression is equivalent to the one given in the beginning of this subsection.

Another way to express Wick polynomials is to decompose the symbol in an orthonormal basis. Let $e_0(s) := \pi^{-\frac{1}{4}} \mathrm{e}^{-\frac{s^2}{2}}$ and $e_j(s) := -\frac{1}{\sqrt{2^j j!}} P_j(s) e_0(s)$, $j \in \mathbb{N}$, where P_j are Hermite polynomials satisfying $\frac{d^j}{ds^j} \mathrm{e}^{-\frac{s^2}{2}} = (-1)^j P_j(s) \mathrm{e}^{-\frac{s^2}{2}}$. The family of Hermite functions

$$\{e_J := \prod_{i=1}^{d} e_{j_i}, J \in \mathbb{N}^d\}$$

is an orthonormal basis of $L^2(\mathbb{R}^d)$. Let $w \in \mathcal{B}(\mathcal{K} \otimes \Lambda^p(\mathfrak{h}), \mathcal{K} \otimes \Lambda^q(\mathfrak{h}))$, then we have

$$Wick(w) = \sum_{I,J} \lambda_{I,J}\ b^*(e_{I_1})...b^*(e_{I_q}) \times b(e_{J_p})...b(e_{J_1}),\ I_i, J_i \in \mathbb{N}^d. \tag{2.3}$$

$$\lambda_{I,J} = \langle w, e_{I_1} \wedge ... \wedge e_{I_q} \otimes e_{J_1} \wedge ... \wedge e_{J_p} \rangle \in \mathcal{B}(\mathcal{K}). \tag{2.4}$$

The r.h.s. in (2.3) is weakly convergent for all vectors in $\mathcal{K} \otimes \Lambda^{\text{fin}}(\mathfrak{h})$.

Boundedness of Wick polynomials. We recall a well known estimate [11] implying the boundedness of Wick monomials for certain class of kernels. We define $S_2(\mathbb{R}^{d(p+q)}, \mathcal{B}(\mathcal{K}))$ to be the Banach space of functions $w \in C^2(\mathbb{R}^{d(p+q)}, \mathcal{B}(\mathcal{K}))$ satisfying

$$\sum_{|\alpha|+|\beta| \le 2d(p+q)} \sup_{k,k^{'}} \left\| (k, k^{'})^{\alpha} D^{\beta}_{k,k^{'}} w(k, k^{'}) \right\|_{\mathcal{B}(\mathcal{K})} < \infty,$$

where α, β are multindices in $\mathbb{N}^{d(p+q)}$.

Theorem 2.3. *Let $w \in \mathcal{B}(\mathcal{K} \otimes \Lambda^p(\mathfrak{h}), \mathcal{K} \otimes \Lambda^q(\mathfrak{h}))$. We have*

$$\left\| Wick(w) \right\| \leq c \left\| w(k_1, ..., k_p, k_1^{'}, ..., k_q^{'}) \right\|_{S_2(\mathbb{R}^{d(p+q)}, \mathcal{B}(\mathcal{K}))}.$$

Proof. Using the decomposition (2.3), we see that $||Wick(w)|| \leq \sum_{I,J} ||\lambda_{I,J}||_{\mathcal{B}(\mathcal{K})}$. Since the basis $\{e_J\}$ is formed of Hermite functions which are eigenvectors of the harmonic oscillator $\Theta_h := \frac{1}{2}\sum_{i=1}^{d} -\frac{d^2}{ds_i^2} + s_i^2 - 1$ and using Hölder inequality, we can write

$$\begin{aligned} \sum_{I,J} ||\lambda_{I,J}||_{\mathcal{B}(\mathcal{K})} &\leq \sum_{I,J} ||\lambda_{I,J}||_{\mathcal{B}(\mathcal{K})} \prod_{k=1}^{d(p+q)} (1+i_k) \prod_{k=1}^{d(p+q)} (1+i_k)^{-1}, \\ &\leq \left(\sum_{I,J} ||\lambda_{I,J}||^2_{\mathcal{B}(\mathcal{K})} \prod_{k=1}^{d(p+q)} (1+i_k)^2 \right)^{\frac{1}{2}} \left(\sum_{I,J} \prod_{k=1}^{d(p+q)} (1+i_k)^{-2} \right)^{\frac{1}{2}}, \\ &\leq c || \prod_{k=1}^{p+q} (1+\Theta_h) w ||_{L^2(\mathbb{R}^{d(p+q)}, \mathcal{B}(\mathcal{K}))}. \end{aligned}$$

The norm appearing in the last inequality is equivalent to the Schwartz norm claimed in the theorem (see e.g. [20, exemple 7]). ∎

Hence, we conclude that the map $S_2(\mathbb{R}^{d(p+q)}, B(\mathcal{K})) \ni w \to Wick(w) \in \mathcal{B}(\mathcal{K} \otimes \Lambda(\mathfrak{h}))$ is continuous. It is also equivalent to the continuity of the map which to a sequences $\lambda_{I,J}$ with values in $\mathcal{B}(\mathcal{K})$ satisfying $\sum_{I,J} ||\lambda_{I,J}||^2_{\mathcal{B}(\mathcal{K})}$ $\prod_{k=1}^{d(p+q)} (1+i_k)^2 < \infty$, corresponds a Wick monomial with respect to the decomposition (2.3).

2.3. CAR algebra

Let (L, S) be an orthogonal space (i.e: a real topological vector space endowed with a continuous symmetric bilinear form S on L). A CAR algebra $\mathfrak{U}(\mathfrak{h})$ over (L, S) is a pair $(\mathcal{D}, \Phi_\pi)$ consisting on a Hilbert space $\mathcal{D}$ and a linear map $L \ni h \to \Phi_\pi(h) \in \mathcal{B}(\mathcal{D})$ into self-adjoint bounded operator and satisfying

$$\{\Phi_\pi(h), \Phi_\pi(g)\} = S(h,g)\mathbb{1}. \qquad \textit{(Clifford relation)}$$

We assume that (L, S) is equipped with a complex structure consisting on a conjugation $\mathcal{J} : L \to L, \mathcal{J}^2 = \mathbb{1}$ compatible with the symmetric bilinear form S in the following sense:

i) $S(h, \mathcal{J}g) + S(\mathcal{J}h, g) = 0,$
ii) $S(h, h) > 0$, for all $h = 0$.

This allows us the construction of creation-annihilation operator

$$\begin{aligned} B_\pi(h) &:= \frac{1}{\sqrt{2}}(\Phi_\pi(h) - \mathrm{i}\Phi_\pi(\mathcal{J}h)), \\ B_\pi^*(h) &:= \frac{1}{\sqrt{2}}(\Phi_\pi(h) + \mathrm{i}\Phi_\pi(\mathcal{J}h)). \end{aligned}$$

$B_\pi(h), B_\pi^*(h)$ are operators satisfying canonical anti-commutation relation as in (2.1)-(2.2), with the inner product $(h|g) = S(h,g) + iS(h, \mathcal{J}g)$. Furthermore, the Pauli principle is satisfied and we have $||B_\pi^*(h)|| = ||B_\pi(h)|| = ||h||$. For more details, one can see [6], [9].

In the sequel we denote by $\mathfrak{U}(\mathfrak{h})$ the CAR algebra over $\mathfrak{h} := L^2(\mathbb{R}^d)$ endowed with the bilinear form $\mathrm{Re}(.,.)$. As in Subsect. 2.2, we can introduce bounded Wick polynomials as elements of the algebra $\mathfrak{U}(\mathfrak{h})$. For this it is enough to use the decomposition (2.3) with $B_\pi^*(.), B_\pi(.)$ instead of $b^*(.), b(.)$ and a symbol $w \in \mathcal{B}(\Lambda^p\mathfrak{h}, \Lambda^q\mathfrak{h})$.

$$Wick_\pi(w) := \sum_{I,J} \lambda_{I,J}\ B_\pi^*(e_{I_1})...B_\pi^*(e_{I_q}) \times B_\pi(e_{J_p})...B_\pi(e_{J_1}),$$

where

$$\lambda_{I,J} := \langle w, e_{I_1} \wedge ... \wedge e_{I_q} \otimes e_{J_1} \wedge ... \wedge e_{J_p}\rangle \in \mathcal{B}(\mathcal{K}), \quad \text{and} \ \sum_{I,J} \left\|\lambda_{I,J}\right\|_{\mathcal{B}(\mathcal{K})} < \infty.$$

2.4. The spin fermion system

In this subsection we describe in detail the considered model. In particular, we specify the required hypothesis under which we construct the counterexample in Thm. 2.7.

The free dynamic. Let $\mathcal{K}$ be a complex Hilbert space (*the small system*). In all the sequel we assume

$$(H0) \quad \mathcal{K} \text{ finite dimension.}$$

We are considering the case $\mathfrak{h} := L^2(\mathbb{R}^d, dk)$ in the momentum representation.

Let $\mathcal{U}$ be the $\mathbb{C}^*$-algebra $\mathcal{B}(\mathcal{K}) \otimes^* \mathfrak{U}(\mathfrak{h})$ endowed with the $\mathbb{C}^*$-norm defined by

$$\|\sum_i A_i \otimes B_i\|_* = \|\sum_i \pi_1(A_i) \otimes \pi_2(B_i)\|,$$

where π_1, π_2 are faithful representations of $\mathcal{B}(\mathcal{K})$ and $\mathfrak{U}(\mathfrak{h})$. This norm was been introduced by Turumaru in [24]. It is independent from π_1, π_2 and it is the smallest $\mathbb{C}^*$-norm. Moreover it satisfies the cross norm property $\| A \otimes B \| = \|A\| \times \|B\|$ and within the hypothesis $(H0)$ satisfied, it is the only one $\mathbb{C}^*$-norm over $\mathcal{U}$, see [22].

Let $\omega \in C(\mathbb{R}^d, \mathbb{R})$ be the dispersion relation of fermions. We assume that ω satisfies the following hypothesis.

$$(H1) \quad \begin{cases} \nabla\omega \in L^\infty(\mathbb{R}^d), \\ \nabla\omega(k) \neq 0, \text{ for } k \neq 0, \\ \lim\limits_{k\to\infty} \omega(k) = +\infty, \\ \inf\limits_k \omega(k) = \omega(0) =: m > 0. \end{cases}$$

The free dynamic is defined by

$$\tau_t^0(A \otimes B_\pi^\sharp(h)) := \mathrm{e}^{\mathrm{i}tK} A \mathrm{e}^{-\mathrm{i}tK} \otimes B_\pi^\sharp(\mathrm{e}^{\mathrm{i}t\omega}h).$$

The sharp mains that the formula is valid with $*$ and without. τ_t^0 extends by linearity to $\mathcal{B}(\mathcal{K}) \otimes_{\mathrm{Alg}} \mathfrak{U}(\mathfrak{h})$ and then by density to $\mathcal{U}$.

Perturbation. Let P be an element in $\mathcal{U}$ such that $P^* = P$. Then we can construct the perturbed dynamic τ_t^P respectively to the interaction P by Dyson's series. For this, we recall a general result, see [9, Prop. 5.4.1] and [8, Thm. 3.1.33].

Proposition 2.4. *For every $P = P^* \in \mathcal{U}$ there exists a one-parameter group of $*$-automorphisms τ_t^P defined by the following convergent series*

$$\tau_t^P(A) := \tau_t^0(A) + \sum_{n\geq 1} \mathrm{i}^n \int_0^t dt_1 \int_0^{t_1} dt_2 ... \int_0^{t_{n-1}} dt_n\ [\tau_t^0(P), [...[\tau_t^0(P), \tau_t^0(A)]]]. \tag{2.5}$$

Moreover, there exists a one-parameter family of unitary elements Γ_t^P *in* $\mathcal{U}$ *satisfying*

$$\Gamma_t^P = \mathbb{1} + \sum_{n\geq 1} \mathrm{i}^n \int_0^t dt_1 \int_0^{t_1} dt_2 \ldots \int_0^{t_{n-1}} dt_n \tau_{t_n}^0(P)\ldots\tau_{t_1}^0(P),$$
$$\tau_t^P(A) = \Gamma_t^P \, \tau_t^0(A) \, \Gamma_t^{P^*},$$

with the co-cycle relation:

$$\Gamma_{t+s}^P = \Gamma_t^P \, \tau_t^0(\Gamma_s^P).$$

We notice that in a recent work [15] the above construction was been extended to more general perturbations.

Let $\mathcal{H}$ denotes the Hilbert space of the small system interacting with the reservoir in the Fock representation i.e:

$$\mathcal{H} := \mathcal{K} \otimes \Lambda(\mathfrak{h}).$$

When τ_t^0 is acting on this Hilbert space it is unitary implemented in the following way,

$$\tau_t^0(A \otimes B) = \mathrm{e}^{\mathrm{i}tH_0} A \otimes B \mathrm{e}^{-\mathrm{i}tH_0},$$
$$H_0 := K \otimes \mathbf{1} + \mathbf{1} \otimes \mathrm{d}\Gamma(\omega), \text{ acting on } \mathcal{H}.$$

We mention that H_0 is essentially self-adjoint on $\mathcal{D}(K) \otimes \Lambda^{\mathrm{fin}}(C_0^\infty(\mathbb{R}^d))$. Thanks to the perturbation expansion in Prop. 2.5, we conclude that the perturbed dynamic is also unitary implemented, see [9, Corollary 5.4.2] for the proof. We have

$$\tau_t^P(A \otimes B) = \mathrm{e}^{\mathrm{i}tH_\mu} A \otimes B \mathrm{e}^{-\mathrm{i}tH_\mu},$$
$$H_\mu := H_0 + \mu V, \text{ acting on } \mathcal{H},$$

where V denotes the element $\frac{P}{\mu}$ in the Fock representation and μ is the coupling parameter. The Hamiltonian H_μ is self-adjoint with domain $\mathcal{D}(H_\mu) = \mathcal{D}(H_0)$.
We consider now the perturbation having the following form,

$$P := \mu \sum_{p,q\in\Xi} Wick_\pi(w_{p,q}), \quad \text{where } \Xi := \{(p,q) \in \mathbb{N}^2 \mid p+q \text{ even } \},$$
$$\text{and } w_{p,q} \in \mathcal{B}(\mathcal{K} \otimes \Lambda^p(\mathfrak{h}), \mathcal{K} \otimes \Lambda^q(\mathfrak{h})), \text{ satisfying } w_{p,q}^* = w_{q,p}.$$

One can assume

$$(I0) \quad \sum_{p,q\in\Xi} \sum_{I,J} \| \langle w_{p,q}, e_I \otimes e_J \rangle \|_{\mathcal{B}(\mathcal{K})} < \infty.$$

The hypothesis $(I0)$ ensure that $P \in \mathcal{U}$ and hence, the perturbed dynamic is well defined. In the Fock representation, we have $V = \sum_{p,q\in\Xi} Wick(w_{p,q}) \in \mathcal{B}(\mathcal{K} \otimes \Lambda(\mathfrak{h}))$ and the Hamiltonian H_μ is well defined self-adjoint operator on $\mathcal{H}$. But, we will assume et slightly stronger condition for V which have the advantage to be more convenient to check since it involves kernel norms of the symbols $w_{p,q}$ rather than the operators norm. Namely,

$$(I0^{'}) \quad \sum_{p,q\in\Xi} \|w_{p,q}\|_{S_2(\mathbb{R}^{d(p+q)}, \mathcal{B}(\mathcal{K}))} < \infty.$$

In Sect. 3 we will recall a result which gives a description of the nature of the spectrum of H_μ. It holds under an assumption of locality on the interaction, which is

$$(I1) \qquad \sum_{p,q\in\Xi}\sum_{i=1}^{p+q} ||\mathbf{1}_{[R,\infty[}(|x_i|)\, w_{p,q}||_{S_2(\mathbb{R}^{d(p+q)},\mathcal{B}(\mathcal{K}))} \in o(R^0).$$

2.5. Results

The Møller morphisms are the equivalent of wave operator for scattering theory on $\mathbb{C}^*$-algebras framework, denoted by

$$\gamma^{\pm}(A) := \lim_{t\to\pm\infty} \tau^P_{-t}\circ\tau_t(A),\ A\in\mathcal{U}.$$

we recall the definition for L^1-asymptotically abelian dynamical system.

Definition 2.5. *A $\mathbb{C}^*$-dynamical system $(\mathfrak{U},\tau)$ is said to be $L^1(\mathfrak{U}_0)$-asymptotically abelian if*

$$\int_{-\infty}^{+\infty} ||[A,\tau_t(B)]||\, dt < \infty$$

for all A, B in the norm-dense $*$-subalgebra $\mathfrak{U}_0$.

If one assume that $(\mathcal{U},\tau^0_t)$ satisfies the $L^1(\mathcal{U})$-asymptotically abelian condition, then using a Cook method we can prove the existence of the Møller morphisms, see [9, Prop. 5.4.10].

One can relax the L^1-asymptotic abelianness condition to a less stronger one appearing in [19]. We say that a $\mathbb{C}^*$-dynamical system $(\mathfrak{U},\tau)$ is L^1-integrable relatively to the perturbation $P\in\mathfrak{U}$ iff

$$\int_{-\infty}^{+\infty} ||[P,\tau_t(A)]||dt < \infty$$

for all A in a dense *-subalgebra $\mathfrak{U}_0$. The above condition leads also to the existence of Møller morphism.

Before giving the result on the non-existence of the Møller morphism and therefore the non faithfulness of the L^1-asymptotic abelianness, we recall a result due to Robinson [19, Prop. 5.4.10] which is the motivation behind the introduction of the later notion.

Theorem 2.6. *Let $(\mathcal{U},\tau)$ be an L^1-asymptotically abelian $\mathbb{C}^*$-dynamical system. The Møller morphism*

$$\gamma^{\pm} = \lim_{t\to\mp\infty} \tau^P_{-t}\circ\tau_t(A),$$

exists for all $A\in\mathcal{U}$ and $P=P^*\in\mathcal{U}$. Moreover
$i)$ The dual $\gamma\pm*$ maps τ^P invariant states into τ-invariant states.
$ii)$ $\gamma^{\pm*}$ maps (τ^P,β)-KMS states into (τ,β)-KMS state and extremal (τ^P,β)-KMS states into extremal (τ,β)-KMS state.

Our main theorem is the following one, it uses hypothesis $(FH0)-(FH2)$ stated in Subsect. 3.1.

Theorem 2.7. *Assuming hypothesis $(H0)-(H1),(I0'),(FH0)-(FH2)$, the møller morphism does not exist and consequently the property $L^1(\mathcal{U})$-asymptotic abelianness and relatively $L^1(\mathcal{U})$-asymptotic abelianness are not satisfied for the spin-fermion dynamic.*

The proof figure in Sect.4.

3. SPECTRAL ANALYSIS

In this section we study the spectral properties of the spin-fermion model. This will be a tool to prove the non-existence of the Møller morphism. We start recalling a result on the existence of a ground state far from the essential spectrum with a gap equal to m, Thm. 3.8. In Subsect. 3.1, using Aguilar-Combes complex scaling technique, we prove that all embedded eigenvalues outside thresholds of the free dynamic will dissolve in the continuum spectrum turning into resonances poles. We derive the Fermi golden rule, giving the expectation on the imaginary part of those poles. This question was been treated by several authors in many different situations and using different techniques, for example the massive, massless case zero temperature, positive temperature and using complex translation or complex dilation deformation. We cite some of this works [17], [12], [13], [4], [10].

Next, a HVZ theorem describing the qualitative aspect of the spectrum of H, [2].

Theorem 3.8. *Assume* $(H0)-(H1)$ *and* $(I1)$ *satisfied, then*

$$\sigma_{\rm ess}(H)=[\inf\sigma(H)+m,+\infty[.$$

Moreover, $\inf\sigma(H)$ *is a discrete eigenvalue.*

3.1. Fermi Golden Rule

In this subsection we study perturbation of eigenvalues of H_0 when a small coupling interaction is turned on. We follow essentially the paper of [17]. The main ingredients are regular perturbation theory and complex scaling technique. We define the dilation unitary group $U(\theta)$ for $\theta\in\mathbb{R}$ acting on the Fock space as follows:

$$\begin{aligned}&U(\theta):\Lambda(\mathfrak{h})\to\Lambda(\mathfrak{h}),\\ &U(\theta)\psi^n(k_1,...,k_n):=\mathrm{e}^{nd\frac{\theta}{2}}\,\psi^n(\mathrm{e}^{\theta}k_1,...,\mathrm{e}^{\theta}k_n),\ \psi^n\in\Lambda^n(\mathfrak{h}).\end{aligned}$$

Let $\mathcal{A}(\delta)$ denotes the sector $\{z\in\mathbb{C};|\arg(z)|\le\delta\}$ and $\mathbb{C}_\delta$ the strip $\{\theta:|\mathrm{Im}\theta|<\delta\}$. Along this subsection we assume that hypothesis $(H0)$, $(H1)$ and $(I0^{'})$ are satisfied. The operator K has eigenvalues

$$E_1<...<E_n,$$

where each eigenvalue count for multiplicity one. We denote the thresholds set by

$$\tau:=\{E_j+nm,n\in\mathbb{N}^*\}.$$

Let give now the main hypothesis needed for results in this subsection. We assume

$$(FH0)\qquad E_j\notin\tau,\ \text{for}\ j=1...n.$$

The dispersion relation should satisfies:

$(FH1)$ The map $\theta\to\omega_\theta(k):=\mathrm{e}^{d\frac{\theta}{2}}\omega(\mathrm{e}^{\theta}k)$ has analytic continuation into the strip $\mathbb{C}_\delta$ such that:

$$\begin{cases} 0 < c_1 \leq \frac{\omega_\theta(k)}{\omega(k)} \leq c_2, \text{ uniformly in compact subsets of } \mathbb{C}_\delta. \\ \text{If } \mathrm{Im}\theta < 0, \text{ then } -\delta < \arg(\omega_\theta) \leq 0. \end{cases}$$

We recall that V is given by a Wick polynomial with symbols $w_{p,q} \in S_2(\mathbb{R}^{d(p+q)}, \mathcal{B}(\mathcal{K}))$ and satisfying hypothesis $(I0^{'})$. For $\theta \in \mathbb{R}$, we define operator $w_{p,q}^\theta \in \mathcal{B}(\mathcal{K} \otimes \Lambda^p(\mathfrak{h}), \mathcal{K} \otimes \Lambda^q(\mathfrak{h}))$ as the operator with the kernel $\mathrm{e}^{d(p+q)\frac{\theta}{2}} w_{p,q}(\mathrm{e}^\theta k_1, .., \mathrm{e}^\theta k_p, \mathrm{e}^\theta k_1^{'}, .., \mathrm{e}^\theta k_q^{'})$.
Let now deal with analytic dilation for the interaction. We assume the following:

$(FH2)$ The maps $\theta \to w_{p,q}^\theta \in S_2(\mathbb{R}^{d(p+q)}, \mathcal{B}(\mathcal{K}))$ has analytic continuation in the sector $\mathbb{C}_\delta$ for all $p, q \in \Xi$ and such that

$$\sum_{p,q\in\Xi} \sup_{|\theta|<\delta} ||w_{p,q}^\theta||_{S_2(\mathbb{R}^{d(p+q)}, \mathcal{B}(\mathcal{K}))} < \infty$$

and the map $\theta \to w^\theta := \sum_{p,q\in\Xi} w_{p,q}^\theta$ is differentiable in the complex domain $\mathbb{C}_\delta$ with respect to the above norm.

We define the family of operators $H_0(\theta), \theta \in \mathbb{C}_\delta$:

$$H_0(\theta) := K \otimes \mathbf{1} + \mathbf{1} \otimes \mathrm{d}\Gamma(\omega_\theta), \text{ acting on } \mathcal{D}(H_0).$$

They satisfy the following proposition.

Proposition 3.9. *Assume $(FH1)$ holds, we have:*
$i)$ *$\mathbb{C}_\delta \ni \theta \to H_0(\theta)$ is an analytic family of type (A).*
$ii)$ *$H_0(\theta)$ is strictly m-sectorial with semi-angle δ_0.*
$iii)$ *$\sigma_{\mathrm{ess}}(H_0(\theta)) = \{E_j + nm + \omega_\theta(k), k \in \mathbb{R}^d\}$, $\sigma_{\mathrm{pp}}(H_0(\theta)) = \{E_j\}$.*

Proof. Using the fact that ω_θ is equivalent to ω, we see that $H_0(\theta)$ is a family of closed operators with the same domain $\mathcal{D}(H_0)$ for all $\theta \in \mathbb{C}_\delta$. It is enough to prove that $H_0(\theta)\psi, \psi \in D(H_0)$ is differentiable in the complex domain $\mathbb{C}_\delta$, to show $i)$. In fact it suffices to show differentiability only for $\psi \in \mathcal{K} \otimes \Lambda^{\mathrm{fin}}(C_0^\infty(\mathbb{R}^d))$. We have

$$\begin{aligned} &H_0(\theta) - H_0(\theta^{'}) - (\theta - \theta^{'})\mathrm{d}\Gamma(\frac{\mathrm{d}}{\mathrm{d}\theta}\omega_\theta)\psi^n(k_1, ..., k_n) = \\ &\sum_{i=1}^n \omega(\mathrm{e}^\theta k_i) - \omega(k_i) - (\theta - \theta^{'})\frac{\mathrm{d}}{\mathrm{d}\theta}\omega_\theta(k_i)\psi^n(\mathrm{e}^\theta k_1, ..., \mathrm{e}^\theta k_n). \end{aligned}$$

Hence the analytic continuation of ω_θ, hypothesis $(FH1)$, complete the proof of $i)$.
$ii) - iii)$ they are simple and left to the reader. ▌

We denote The dilated interaction $V(\theta), \theta \in \mathbb{C}_\delta := Wick(w^\theta)$, it satisfies the next proposition.

Proposition 3.10. *Under the assumption $(FH2)$ satisfied, we have the map $\theta \to V(\theta)$ is an holomorphic family of bounded operators.*

Proof. It is enough to show that in the complex domain $\mathbb{C}_\delta$ the map $\theta \to V(\theta)$ is differentiable. Using estimate in Thm. 2.3, we obtain

$$\begin{aligned}&\big\|V(\theta) - V(\theta+h) - hWick(\sum_{p,q\in\Xi} \frac{d}{d\theta} w^\theta_{p,q})\big\| \leq \\ &\sum_{p,q\in\Xi} \sup_{|\theta|<\delta} \big\|w^\theta_{p,q} - w^{\theta+h}_{p,q} - h\frac{d}{d\theta} w^\theta_{p,q}\big\|_{S_2(\mathbb{R}^{d(p+q)},\mathcal{B}(\mathcal{K}))}.\end{aligned} \tag{3.1}$$

Now the hypothesis $(FH2)$ ensure the limit in the r.h.s. of (3.1). This gives the differentiability. ▮

We have the following proposition as consequence of the above ones.

Proposition 3.11. *Assume (FH1)-(FH2) hold, we have:*
$i)$ $\mathbb{C}_\delta \ni \theta \to H_\mu(\theta)$ *is an analytic family of type (A).*
$ii)$ *For* $\theta \in \mathbb{C}_\delta$ *and* $\theta' \in \mathbb{R}$, *we have*

$$U(\theta')H_\mu(\theta)U(\theta')^{-1} = H_\mu(\theta' + \theta).$$

Proof. Using the fact that $V(\theta)$ is H_0-bounded and the Kato-Rellich theorem, we see that $H_\mu(theta), \theta \in \mathbb{C}_\delta$ is closed operator on $\mathcal{D}(H_0)$. The analyticity follows from those of $H_0(\theta)$ and $V(\theta)$. The second point is simple computation. ▮

Theorem 3.12. *Assume* $(H0)-(H1)$, $(FH0)-(FH2)$ *satisfied.*
$i)$ *Then there exists for* μ *small enough a dense subset* $\mathcal{D}$ *in* $\mathcal{H}$ *such that the following map*

$$z \to (\phi, (H_\mu - z)^{-1}\psi), \quad \psi, \phi \in \mathcal{D},$$

has a meromorphic continuation from the upper half-plane into regions $\{z : |z - E_j| < c_j(\mu)\}$.
$ii)$ *The Rayleigh-Schrödinger series for perturbed eigenvalues had a non zero convergence radius and it is given by*

$$E_j(\mu) = E_j + \mu^2\Gamma_j + O(\mu^3),$$

where the time life of resonances is given by

$$\begin{aligned}\Gamma_j \;=\; & -\mathrm{i}\pi \sum_{i<j}^{n} \sum_{q\in\sigma_{j,i}} |\langle w_{0,q}\psi_j, \psi_i\rangle_{\mathcal{K}}|^2(k_0^1, ..., k_0^q) - \sum_{i<j}^{n} \sum_{q\in\sigma_{j,i}} \mathrm{PV}\int \frac{|\langle w_{0,q}(k_1, ..., k_q)\psi_j, \psi_i\rangle_{\mathcal{K}}|^2}{E_i - E_j + \sum_{\ell=1}^q \omega(k_\ell)} dk \\ & + \sum_{i\geq j}^{n} \sum_{q\in 2\mathbb{N}} \int \frac{|\langle w_{0,q}(k_1, ..., k_q)\psi_j, \psi_i\rangle_{\mathcal{K}}|^2}{E_i - E_j + \sum_{i=1}^q \omega(k_i)} dk,\end{aligned}$$

with $\sigma_{j,i} = \{q \in 2\mathbb{N}, qm < E_j - E_i\}$ *and* ψ_i *are eigenvector of* K *for the eigenvalue* E_i.

Proof. The proof follows standard arguments of the complex dilation technique [1], [7], [21]. The spectrum of $H_0(\theta), \theta \in \mathbb{C}_\delta$ is given by Prop. 3.9, where it appears that the eigenvalues $E_j, j = 1...n$, are discrete. Therefore using regular perturbation theory, we know that for small μ there exists $E_j(\mu)$ eigenvalues of $H_\mu(\theta)$ satisfying $\lim_{\mu\to 0} E_j(\mu) = E_j$ and this holds as long as $E_j(\mu)$ does not touch the rest of the spectrum. Moreover, $E_j(\mu)$ is θ independent since it is an analytic function in θ constant on the real line.

Let $\mathcal{D}$ be the set of analytic vectors in the strip $\mathbb{C}_\delta$ of the generator A of the dilation i.e: $\psi \in \mathcal{D}$, $\sum_{n\geq 0}^{\infty} \frac{(\mathrm{i}\theta)}{n!} A^n\psi, \theta \in \mathbb{C}_\delta$ has radius of convergence δ which is equivalent to the fact that $\theta \to U(\theta)\psi, \theta \in \mathbb{R}$ has an analytic continuation to the strip $\mathbb{C}_\delta$. Let $\psi, \phi \in \mathcal{D}$, the map

$$z \to (\phi, (H_\mu - z)^{-1}\psi),$$

is analytic for $\mathrm{Im}z > 0$. For $\theta \in \mathbb{R}$ and $\mathrm{Im}z > 0$, we have

$$(\phi, (H_\mu - z)^{-1}\psi) = (U(\theta)\phi, (H_\mu(\theta) - z)^{-1}U(\theta)\psi),$$

and every term in the r.h.s. has analytic continuation in θ to the strip $\mathbb{C}_\delta$. Now, using the information on the spectrum of $H_\mu(\theta), \mathrm{Im}\theta < 0$, we conclude that the map $z \to (\phi, (H_\mu - z)^{-1}\psi)$ has a meromorphic continuation to regions $B_j := \{z : |E_j - z| < c_j(\mu)\}$ where $c_j(\mu)$ is chosen in such a way that no other part of the spectrum is in the ball B_j except $E_j(\mu)$. This proves $i)$.
$ii)$ holds from Rayleigh-Schrödinger series [18] and computation. Let $\Psi_j \otimes \Omega$ be the eigenvector of H_0 associated with the eigenvalue E_j. We have a convergent series

$$E_j(\mu) = E_j + E_j^{(1)}\mu + E_j^{(2)}\mu^2 + ...,$$

with

$$\begin{aligned} E_j^{(1)} &= \langle \Psi_j \otimes \Omega, V(\theta)\Psi_j \otimes \Omega\rangle = 0, \\ E_j^{(2)} &= \frac{1}{2\mathrm{i}\pi}\oint_{|E_j - z| = \eta_j} \langle \Psi_j \otimes \Omega, [V(\theta)(H_0(\theta) - z)^{-1}]^2\Psi_j \otimes \Omega\rangle dz. \end{aligned}$$

The last term leads to the claimed Γ_j by simple computation. ▮

4. THE MØLLER MORPHISM

In this section we achieve the proof of Thm. 2.7. The main ingredients are the spectral theorem and Fermi golden rule.

Proof of Thm.2.7. Assume that $(\mathcal{U}, \tau_t^0)$ is L^1- asymptotic abelian, then the strong limit $\gamma^\pm = s-\lim_{t\to\pm\infty} \tau_{-t} \circ \tau_t^0$ exists. Let δ_0, (resp. δ) be the generator of τ_t^0(resp. τ_t). They are closed inner derivations with $\delta = \delta_0 + \mathrm{i}[P, .]$ and $\mathcal{D}(\delta) = \mathcal{D}(\delta_0)$.

Consider S a self-adjoint operator on $\mathcal{H}$, then $(S \pm \mathrm{i})^{-1}$ are normal operators. The spectral theorem for normal bounded operators provide us with a direct decomposition $\mathcal{H} = \oplus_{n=1}^N, N \in \mathbb{N} \cup \{\infty\}$ and an unitary transformation,

$$\begin{aligned} &\mathcal{J}_n : \mathcal{H}_n \to L^2(\mathbb{R}, d\nu_n), \\ &\mathcal{J}_n(S \pm \mathrm{i})^{-1}\mathcal{J}_n{}^{-1}\phi(\lambda) = (\lambda \pm \mathrm{i})^{-1}\phi(\lambda), \end{aligned}$$

where ν_n are finite Baire measures. Let $A \in \mathcal{U}, A \neq 0$, satisfying $\mathrm{e}^{\mathrm{i}tS}A\mathrm{e}^{\mathrm{i}tS} = \mathrm{e}^{\mathrm{i}tE}A$. Denote, δ_S the generator of the automorphism defined over $\mathcal{U}$ by $\tau_S(B) := \mathrm{e}^{\mathrm{i}tS}B\mathrm{e}^{\mathrm{i}tS}$, $B \in \mathcal{U}$ and assume that the continuous singular spectrum of S is empty. The bounded operator $\mathcal{J}_n A \mathcal{J}_m{}^{-1}$ acting on $L^2(\mathbb{R}, d\nu_m) \to L^2(\mathbb{R}, d\nu_n)$ can be extended to an operator $S(\mathbb{R}) \to S'(\mathbb{R})$, since ν_n are Baire measures. Therefore, it has a kernel representation and the condition satisfied by A becomes

$$\mathrm{e}^{\mathrm{i}t(\lambda-\lambda')}A(\lambda, \lambda') = \mathrm{e}^{\mathrm{i}tE}A(\lambda, \lambda'),\ t \in \mathbb{R},$$

in the essential support of the measure $\nu_m \otimes \nu_n$. This leads to the fact that the above equality is at least non zero for certain n, m and it is valid pointwise. Hence, it corresponds to ν_n, ν_m which are Dirac measures and therefore associated to the pure punctual spectrum of S. We conclude that $\sigma_{\rm pp}(\delta_S) = \{\lambda_i - \lambda_j, \lambda_i, \lambda_j \in \sigma_{\rm pp}(S)\}$. Applying this for H_0, H_μ, we conclude that

$$\sigma_{\rm pp}(\delta_0) = \{E_i - E_j, i, j = 1, ..., n\},$$
$$\sigma_{\rm pp}(\delta) = \{\tilde{E}_i(\mu) - \tilde{E}_j(\mu), \tilde{E}_i(\mu) \in \sigma_{\rm pp}(H_\mu)\}.$$

Let $A_{i,j} \in \mathcal{U}$, denotes

$$A_{i,j} := \gamma^{\pm}(|\psi_i\rangle\langle\psi_j| \otimes \mathbf{1}).$$

We have

$$\tau_t(A_{i,j}) = \mathrm{e}^{\mathrm{i}t(E_i - E_j)} A_{i,j}.$$

This means that

$$\sigma_{\rm pp}(\delta_0) \subset \sigma_{\rm pp}(\delta).$$

One can conclude that we have

$$E_i - E_j = \tilde{E}_i(\mu) - \tilde{E}_j(\mu)$$

This is in contradiction with the Fermi golden rule 3.1. Indeed, taking μ small and for example $E_j > E_1 + m, j = 2...n$, we know from Subsect. 3.1, that those eigenvalues dissolve in the continuous spectrum and therefore H_μ has only one eigenvalue which is the perturbation of E_1 and the rest is absolute continuous.

Conclusion. The progress made in the study of properties of return to equilibrium of quantum dynamical systems, [14], [5], provides us with the idea that a small system interacting with a reservoir tends to return to equilibrium whereas the non-interaction system does not faithful this property. This confirms the physical expectation of improvement of the ergodic properties when the interaction is turned on. In the light of this consideration, definition of stability of states for local perturbations of the dynamics given in [16, D. Kastler, Def.4 (iii)], telling

$$\omega^P(\tau_t^0(A)) \to \omega(A), \ t \to \pm\infty.$$

seems to be unclear since it is in a certain sense return to equlibiruim of the free dynamic rather the interacting one. It is more reasonable to expect that τ_t^0-KMS states relax to τ_t^P-KMS state i.e:

$$\omega(\tau_t^P(A)) \to \omega^P(A), \ t \to \pm\infty. \tag{4.1}$$

Therefore, There is some confusion in what is considered a perturbed dynamic, since the non-interacting system could be seen as a perturbation of the interacting. In this sense condition (4.1), where P states for the interacting dynamic, agrees better with the physical situation. The other critic on the time-dependent approach is the dependence on strong assumptions as L^1-asymptotic abelianness or exitence of Møller morphism, which reduces its range of application.

Nevertheless, the non-stationary approach is an interesting way to study features of the equilibrium states particularly in the zero temperature where we are not provided with the modular theory and it would be interesting to improve the existing results.

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