On the cosmological implications of the Calogero conjecture

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Abstract

We discuss the implications of the Calogero hypothesis on the "cosmic origin of quantization" [1], and of the variation of h with time it implies, for cosmology. Data which with the standard interpretation of redshift measurement imply uniform expansion are to be interpreted, in the light of this conjecture, as corresponding to an accelerating expansion.

1. Introduction.

In a recent paper, Calogero [1] reconsidered the "universal background noise" which, according to Nelson's stochastic mechanics [2, 3, 4] is at the basis of quantum behaviour. Calogero considered the possible *physical* origin of this noise, pointing his attention to gravitational interactions with distant masses in the Universe (a different mechanism is considered in [5]); he argued that in this way one obtains a "prediction" for the Planck constant given by

$$h \approx \alpha G^{1/2} m^{3/2} [R(t)]^{1/2} \tag{1}$$

with α a numerical constant. Here G is the Newton gravitational constant, m is th emass of the hydrogen atom, considered as the basic unit of mass in the Universe, and $R(t)$ represents the radius of the Universe, or better of the part of it accessible to gravitation interactions, at time t.

With the rough estimate $R(t_0) \simeq 10^{28}$ cm for the present value of R and taking $\alpha = 1$, equation (1) gives the remarkable estimate $h \approx 6 \cdot 10^{-26}\,\mathrm{cm}^2\mathrm{g}\,\mathrm{s}^{-1}$.

It should be noted that R varies with time and therefore (unless G and/or m also vary with time and a miracolous cancellation occurs) the Calogero conjecture (1) entails that h is also varying with time.

If we accept that G and m are truly constant (which we do), then it follows that according to Calogero

$$h \equiv h(t) = A \sqrt{R(t)} \tag{2}$$

with A a dimensional constant.

We want to discuss the consequences of this conjecture for cosmology, and in particular on Hubble's law and on the issue of the age of the Universe.

2. Relation between observed frequency and recession speed.

We will consider an expanding Universe and a body moving radially away from us with speed $v = cw$. We know from Relativity that e.m. radiation emitted with frequency $\nu_{\rm em}$ will be observed by us with frequency

$$\nu_{\rm ob} \ = \ \frac{1-w}{\sqrt{1-w^2}} \, \nu_{\rm em} \ . \tag{3}$$

Inverting (3), once we know $\nu_{\rm em}$, w can be measured in terms of $\nu_{\rm ob}$; more precisely, we have

$$w \ = \ \left(\frac{\nu_{\rm em}^2 - \nu_{\rm ob}^2}{\nu_{\rm em}^2 + \nu_{\rm ob}^2} \right) \ . \tag{4}$$

Let us focus, for the sake of concreteness, on a well specific radiation, i.e. the one corresponding to th first spectral line of hydrogen. We know then that

$$\nu_{\rm em} \ = \ \frac{B}{h^3} \tag{5}$$

where, with standard notation, $B = (3/8) m_{\rm e} e^4 \pi^2$. We denote the value of (5) with the known value (according to Calogero, the known present value) of h by ν_0. We write $\nu_{\rm ob} = \beta \nu_0$ and $\nu_{\rm em} = \mu \nu_0$.

According to standard quantum mechanics, $\nu_{\rm em} = \nu_0$ (i.e. $\mu = 1$) and thus (4) reads

$$w \ = \ \frac{1-\beta^2}{1+\beta^2} \ . \tag{6}$$

According to Calogero conjecture, due to the radiation being emitted at different time and thus with a different value of $h(t)$, we have $\nu_{\rm em} \neq \nu_0$ (i.e. $\mu \neq 1$) and thus (4) reads

$$w \ = \ \frac{\mu^2 - \beta^2}{\mu^2 + \beta^2} \ . \tag{7}$$

For the first hydrogen spectral line we would have (with t_0 the present time, $t_{\rm em}$ the emission time)

$$\nu_{\rm em} \ = \ \frac{B}{h^3(t_{\rm em})} \ = \ \frac{B}{h^3(t_0)} \left[\frac{h(t_0)}{h(t_{\rm em})} \right]^3 \tag{8}$$

and therefore

$$\mu \ = \ \frac{\nu {\rm em}}{\nu_{\rm ob}} \ = \ \left[\frac{h(t_0)}{h(t_{\rm em})} \right]^3 \ = \ \left[\frac{R(t_0)}{R(t_{\rm em})} \right]^{3/2} \ . \tag{9}$$

In the following we write, for ease of notation, $\tau = t_0 - t_{\rm em}$, and $\Theta = [R(t_0-\tau)/R(t_0)]$; notice that for a uniformly expanding Universe Θ represents the scale factor $a(t_{\rm em})$, where by definition $a(t_0)=1$. Thus, $\mu = [a(t_{\rm em})]^{-3/2}$.

It should be noted that the Calogero measurement (7) of the recession speed does therefore also depend on τ (which obviously depends itself on the distance of the object which emitted the radiation) and on the evolution law for $R(t)$.

3. Expansion of a flat Universe.

In a static Universe, radiation emitted at a distance r would reach us after a time $\tau = r/c$; in an expanding Universe this relation should be modified to take into account the expansion.

This is usually done (assuming uniform expansion) by considering a static model with a varying scale factor $a(t)$: that is, all distances satisfy $x(t) = a(t)x_0$. In particular, $R(t) = a(t)R_0$; the Θ defined above is just $\Theta = [a(t_{\rm em})]^{-1}$.

The evolution of $a(t)$ is governed by the Einstein equations, which for a homogeneous and isotropic matter-dominated Universe are

$$\left(\frac{\dot a}{a}\right)^2 \ = \ \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{10}$$

where ρ is the mass density of the Universe, Λ the cosmological constant, and k a parameter equal to (1,0,-1) for a (closed, flat, open) universe.

We will assume $\Lambda = 0$ and consider the flat case $k=0$. Notice that $\rho(t) = \rho_0/a^3(t)$ [with $\rho_0 \equiv \rho(t_0)$]. Writing $D_0^2 = (8/3)\pi G\rho_0$, (10) reads then $\dot a(t) = D_0/\sqrt{a(t)}$, which is readily solved by separation of variables to give (the c_i will denote arbitrary constants) $a(t) = [(3/2)(D_0t+c_1)]^{2/3}$. With $D_1 = (3/2)D_0$, $c_2 = (3/2)c_1$, this is rewritten as $a(t) = (D_1t+c_2)^{2/3}$. By definition, $a(t_0)=1$, so that $(D_1t_0+c_2)=1$, which yields $c_2 = 1-D_1t_0$ and hence $a(t) = [1-D_1(t_0-t)]^{2/3}$. We rewrite this final result as

$$a(t_0-\tau) \ = \ (1-D_1\tau)^{2/3} \ , \tag{12}$$

with $D_1 = \sqrt{6\pi G\rho_0} = 3\sqrt{(GM)/(2R_0^3)}$, M the total mass of the Universe and R_0 its present radius.

Let us now come back to the relation between r and $(t_0-t)\equiv\tau$. This is given by

$$\int_{t_0-\tau}^{t_0} \frac{c}{a(t)}\,dt \ = \ r \ . \tag{13}$$

Using (12), an elementary integration gives $(3c/D_1)[1-(1-D_1\tau)^{1/3}] = r$ and therefore

$$(1-D_1\tau) \ = \ \left(1-\frac{D_1}{3c}r\right)^3 \ . \tag{14}$$

4. Hubble-Calogero relation.

According to Hubble law (and experimental observations), there is a linear relation between distance of celestial bodies and their recession speed,

$$v_0 = Hr \ . \tag{15}$$

Here H is the Hubble constant (notice this is constant in space but, in principle, not in time), r is measured by various means including direct parallaxis measurements (as of now, even for quite distant objects, thanks to satellite observations), and v_0 is obtained from red shift measurements in the standard ($\mu = 1$) interpretation, i.e. from (6). Thus, in our notation, Hubble law amounts to

$$w_0 \ = \ \frac{1-\beta^2}{1+\beta^2} \ = \ \frac{Hr}{c} \ . \tag{16}$$

We will write $\delta = Hr/c$; this represents a dimensionless unit of distance, depending on t through H. With this (16) reads simply $w_0 = \delta$. Note that the last relation in (16) represents the actually measured relation. From this we get

$$\beta^2 \ = \ \frac{1-\delta}{1+\delta} \ . \tag{17}$$

Inserting this into (7), using (9), and recalling that $\Theta = a(t_0 - \tau)$, we obtain the modification of Hubble law implied by Calogero conjecture:

$$w_{\rm cal} \ = \ \frac{(1+\delta) - [a(t_0-\tau)]^3(1-\delta)}{(1+\delta) + [a(t_0-\tau)]^3(1-\delta)} \ . \tag{18}$$

This is independent of the model we consider for the evolution of Universe. Adopting the model considered in the previous section, we can make use of (12) for $a(t)$, and of the relation (14) in order to express this in terms of δ. In this way, and with $q = D_1/(3H)$, we obtain

$$w_{\rm cal} \ = \ \frac{(1+\delta) - (1-q\delta)^6(1+\delta)}{(1+\delta) + (1-q\delta)^6(1+\delta)} \ . \tag{19}$$

This can be expanded in δ around $\delta = 0$ to give

$$w_{\rm cal} \ \approx \ (1+3q)\,\delta \ + \ \frac{3}{2}q^2\,\delta^2 \ + \ \dots \ . \tag{20}$$

In terms of v and r, (19) reads

$$v \ = \ Hr \ + \ 3qHr \ + \ \frac{3q^2H^2}{2c}r^2 \ + \dots \ . \tag{21}$$

With the rough estimates $M = 4 \cdot 10^{54\pm 8}$g, $R_0 = 10^{28\pm 2}$cm (also used in [1]) we get $q \approx 1.5 \cdot 10^{2\pm 7}$. This huge error bar only allows for qualitative considerations.

5. Conclusions.

If we assume the Calogero conjecture (2) to hold, then radiation from distant objects would have been emitted with a different value of h and thus with a frequency different from the one usually assumed; in particular, the dominant radiation corresponding to the first spectral line of hydrogen would have been emitted with a frequency $B/h^3(t)$ higher than the present one. Therefore, when we compare the observed frequency of radiation from distant objects, the red shift would be higher than in the standard interpretation. This effect increases, more than linearly, for greater distances.

Thus, Calogero conjecture would imply that the existing observational data of the relation between red shift and distance, even for relatively small distances, should be reinterpreted as showing an accelerating expansion of the Universe.

We stress that this modification of Hubble law could be inferred in particular from data for objects at relatively small distances, i.e. in a region where direct measurement of distance (via parallaxis) is possible; this is therefore not subject to experimental uncertainities of the same nature as those necessarily entering the discussion of data referring to extremely far regions of the Universe.

We have considered the simplest model of expansion of the Universe, corresponding to Einstein equations with $k = \Lambda = 0$, and discussed the modification of Hubble law implied by the Calogero conjecture.

References

[1] F. Calogero, "Cosmic origin of quantization", *Phys. Lett. A* **228** (1997), 335-346

[2] E. Nelson, "Derivation of the Schrödinger equation from Newtonian mechanics", *Phys. Rev.* **150** (1966), 1079-1085

[3] E. Nelson, *Quantum fluctuations*, Princeton University Press 1985

[4] F. Guerra, "Structural aspects of stochastic mechanics and stochastic field theory", *Phys. Rep.* **77** (1981), 263-312

[5] G. Gaeta, "Black body radiation and quantum fluctuations", *Phys. Lett. A* **155** (1991), 73-74

[6] C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation*, Freeman 1973